We introduce the WeBWorK Collaborative Learning and Active Support System (CLASS), an extension to the popular online homework system, WeBWorK, available for any touch-enabled device. WeBWorK CLASS is a powerful tool for use in and out of the classroom which engages students, aids instructors, and allows researchers to explore student thinking. Research enhancements allow us to analyze both quantitative and qualitative data from in-class sessions, online homework, and other assessments. We use the system in a first semester calculus class for ongoing iterations of a design experiment focusing on function composition as it relates to calculus concepts such as the chain rule. Examples from collected data demonstrate the utility of CLASS as a research tool. Further, we describe how an approach to calculus based on differentials was used to teach the chain rule and how students interpret it.

Keywords: Calculus; design experiment; online assignment; WeBWorK

1. Introduction
As online assignment systems\(^1\) gain popularity in undergraduate mathematics courses, the effective incorporation of such systems in classroom teaching and educational research studies addressing students’ development of mathematical concepts needs more attention. Research studies on traditional homework indicate that there is a positive association between assignments and achievement [1]. Students who complete online assignments perform as well as those who complete traditional paper and pencil assignments [2,3]. Compared to traditional assignments, students prefer the immediate feedback on answers from online systems and this contributed to higher persistence rates while solving problems [4]. Moreover, online assignment systems provide students additional learning opportunities by providing problem variations, multiple attempts per question, and help via online tutorials. However, online assignment systems could do more.

Online assignment systems have the potential to provide more information on students’ understanding of concepts. We enhanced the free, open-source online assignment system, WeBWorK\(^2\), to create WeBWorK Collaborative Learning and Active Support System (CLASS). The newly incorporated tools include: the integration of a whiteboard to capture students’ written work (Figure 1), an integrated graphing utility (no need to carry a separate calculator or access an external website), digital work maps (Figure 2), CLASS performance map (Figure 3), and anonymous classroom sharing of student generated work.

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\(^1\) Our use of online assignment system includes other assessment aspects, such as online exams.

\(^2\) More information about WeBWorK can be found at http://webwork.maa.org/

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Lighthouse Delta 2013: The 9\textsuperscript{th} Delta Conference on teaching and learning of undergraduate mathematics and statistics, 24-29 November 2013, Kiama, Australia
CLASS works on all touch-enabled platforms including tablets, ipads, laptops, and phones - technologies students increasingly already own. CLASS can be used for any course that has a WebWorK component. CLASS, together with the existing capabilities of WebWorK, provides an all-in-one system that goes beyond static online assignment systems and creates an interactive in-class teaching tool and collaborative research platform.

CLASS contributes to research efforts in the mathematics classroom by collecting and organizing quantitative data. Furthermore, inclusion of the annotation tool helps analyze qualitative data. We use these capabilities in a design experiment [5], intended to address two general questions:

How does CLASS allow researchers to learn more about students’ learning and understanding of mathematical concepts?

How does CLASS contribute to concept development and student success in calculus?

In this paper we provide results from the first iteration of the project, which focuses on use of the annotation tool. We also share our implementation of CLASS in an ongoing cycle of the project.

As a research tool, CLASS efficiently gathers and processes quantitative and qualitative data that are especially useful for mathematics education researchers and teachers. Since CLASS is able to capture all of a student's work, including blind alleys and backups, the researchers can identify when a student is blindly parroting previous work or when she is incorporating old concepts with new ideas correctly. Identifying such student actions allows researchers to track concept development. Moreover, CLASS’ annotation tool provides a mechanism for researchers to investigate students’ understanding collaboratively. We will provide evidence of how collaborating researchers at institutions across the globe can create codes with descriptions, modify existing codes, and link codes to particular times in students’ solutions.

We begin with a brief review of the literature on online assignment systems and in-class technology used to promote engagement in class. The subsequent discussion of technology-supported social constructivism for learning provides the framework for the design experiment, and precedes the description of our design experiment, data, analysis, and results.
2. Background

Students are frequently engaged with learning tasks that take place outside of the classroom, commonly known as homework. It is the expectation that these activities help students understand and master the content of the course. Many research studies have explored the relationship between homework and achievement in courses [1,6,7]. Even though the contribution of each homework factor to achievement is not clear, there is a consensus that assigning homework helps students learn, especially at the secondary and post-secondary levels [1,6-8]. Time spent on homework has shown to be positively correlated with student achievement in a course [1,6]. Furthermore, grading can provide the instructor with a feeling of how the class is learning.

Studies that focus on the impact of online homework on achievement indicate that students who complete online homework assignments perform as well as students completing traditional assignments [2,3]. Moreover, students complete online homework at a higher rate than traditional paper and pencil assignments [4]. Most commercially available online homework systems collect students’ entered answers and provide a basic quantitative analysis reporting the number of students with correct and incorrect attempts to each question. Such information helps instructors quickly identify common troublesome questions and could be used as formative assessment tool [9].

CLASS provides all these benefits and additional information. Similar to tools developed within the SAIL-M project [9], CLASS provides a visual representation of students’ overall performances. However, different from tools in the SAIL-M project, each student’s written solutions can be viewed and replayed through the CLASS digital work map (Figure 2).

![Figure 2. Digital work map.](image)

The CLASS performance map (Figure 3) allows instructors to examine success rates on a chosen set of related questions and the associated students’ answers to see how students understand a concept [10]. For example, various questions in an assignment might focus on different aspects of a particular concept, such as symbolic manipulation, use of different representations, word problem. An instructor could create a CLASS performance map, sorting correct and incorrect answers to each of these different questions to see a possible development of a concept by different categories of students. See Figure 3 for an example based on function composition problems.

![Figure 3. CLASS performance map.](image)
The CLASS performance map can be used to identify areas where the problem solving process breaks down for students and students’ written solutions viewed in the CLASS digital map provides more details of this process. Such instances could be valuable to structure classroom discourse. Reform movements in mathematics education highlight the importance of communication about the process of problem solving and reasoning in classrooms. [11,12] Such shifts call for altered interactions between students and teachers in the classroom. Research has shown that students learn more effectively when they are actively engaged with the material [13-15]. It is critical to integrate tools such as CLASS that promote such learning environments and support student reasoning, communication, and problem solving skills.

Many researchers have suggested that the use of technology in classrooms should contribute to student learning. Graphing calculators and audience response systems (clickers) have been used to promote student thinking [16] and have students interact with the teacher, but the technology has not moved substantially beyond these capabilities. Donovan and Loch [14] used pen-enabled screens to provide opportunities for students to solve problems in groups, give and receive feedback on problems during class time, and become more active learners. Balacheff and Kaput [17] pointed out that the computer-based learning environments in mathematics are powerful not only in their impact on daily practices but also in their contribution to the deeper development of knowledge of individuals. Pea [18] suggested technologies should help students take control of their own learning by supporting the development of skills like problem posing, flexible strategies, creative thinking, summarization, and cooperative problem solving. We will illustrate how we leverage CLASS features to engage students with mathematics.

3. Theoretical Perspective
Our design experiment is structured around social constructivist ideas [19-21]. We are interested in the process of growth and change rather than just the product of development. An important tenet of this theoretical framework is that mental processes are mediated by tools and signs such as language, writing, technological tools, and so on. As students actively engage with tools in the classroom, their experience is internalized and executed on an inner, mental plane. Vygotsky [20, 21] states this incorporation of tools into human action presents new “functions” associated with the tools and alters mental processes. Hence, incorporating technological tools like computers in class affects the nature of the human activity, and can, for example, change the goal of the “learning” activity as well as contribute to the development of psychological tools (signs). Specifically regarding the effect of using computers in the classroom, Moreno-Armella & Sriraman [22] stated, “[A computer] is, simultaneously, a tool that can affect the human activity… and the cognition of the agent user (reorganizing of her ideas). In other words, the computer is externally oriented and at the same time internally oriented.” (p.223) CLASS is the primary tool we are using to restructure the learning experience of the student both in and out of the classroom.

Collectively, the ideas from social constructivist perspectives suggest looking deeper into the construct of the zone of proximal development [21,p. 86], which requires re-evaluation of the role of imitation in learning. It is well documented that students prefer to learn from examples [23-27]. However, evidence suggests this strategy is not as successful as one may hope [23-27]. Silver and Marshall [26] indicated that while experts focus on structure, novices tend to focus on superficial features when attempting to solve word problems. As Boesen et al. [27] reported, most students (86%) successfully solve tasks using imitative reasoning when the new task shares similar surface features with the ones from textbooks or class examples. However, when these surface features are absent, this rate
significantly drops (19%). Capitalizing on information available through CLASS, the researcher can efficiently identify patterns of example imitation on problems similar to in-class and textbook examples. More importantly, CLASS provides mechanisms to compare the in-class and out-of-class work of students on similar tasks which do or do not share superficial features.

4. Method

Our project is a design experiment of a “learning ecology” [5, p.9] as we aim to develop the multiple features of CLASS and study how they function together to support learning in the calculus classroom. By their very nature, design experiments require the collection of large, longitudinal data sets through successive iterations. The data includes products of learning (student work, tests, etc.) as well as data collected from classroom discourse, interviews, social interactions, inscriptions and notations, and other tools. Each iteration of data collection and analysis leads to revisions and testing in the subsequent cycles of the design experiment which make it suitable for testing classroom innovations. Our research team, including the instructor of the course, collaboratively collected and analyzed data, made revisions to CLASS and its use.

4.1 Design Experiment Cycles

During the planning of our design experiment we enhanced the WeBWorK system to create WeBWorK CLASS. After the creation of CLASS, it was first implemented as an online assessment and research tool in a first semester calculus course during the Fall 2010 semester (iteration 1). Our focus in this particular course was the students’ development of calculus concepts that use function composition. This narrow focus was intentional and could be supported by one of the goals of design experiment methodology: “design experiments are conducted to develop theories,... [T]hese theories are relatively humble in that they target domain-specific learning processes.” [5, p.9]

Iteration 2 implemented CLASS in the classroom to capture detailed students’ interactions with each other. In this iteration, the role of CLASS expanded from primarily out of class assessment into the classroom learning experience, and built upon the data analysis from iteration 1. The focus of data collection still revolved around concepts related to function composition.

4.2 Why Function Composition

Historically, calculus is a gateway course for STEM majors. Without achieving success in calculus, STEM majors often move on to other disciplines. Success in calculus can be linked to students’ understanding of the concept of function [28, 29]. The concept of function is known to be difficult for students and slow to develop [30]. While there is fairly extensive literature about student understanding of function, there is little that focuses explicitly on student understanding of function composition. Being able to work with functions in different contexts is particularly problematic for students, especially when dealing with function composition [31]. Having a robust understanding of function composition is necessary for success in calculus as this idea permeates the curriculum. One of the critical units where this concept is foundational is the chain rule and its applications. To understand students’ concept development of function composition and related calculus concepts, we collected students’ written solutions and answers to questions related to this topic throughout a first semester calculus course in both iterations.
4.3 Data Collection

During iteration 1, data was collected from a first semester calculus course taught by one of the authors at an institution that provided tablet technology to all students. The collected data set included all 34 students’ work on pretests3 (Precalculus Concept Assessment instrument4 - PCA, [28]), a sequence of seven CLASS quizzes, online assignments, four midterms, and the final exam throughout the 15-week semester. Students were informed about the data collection and given code numbers prior to any coding to keep their identity anonymous.

Each quiz in the sequence consisted of five questions, two of which involved an aspect of function composition. Students had up to three attempts to answer each quiz question correctly. Each student wrote their solution using digital ink on the whiteboard (Figure 1) integrated into CLASS. This system, further described in [10], recorded student actions and prevented a student’s submission until they had provided enough work on the whiteboard. The whiteboard immediately provided the student with her previous work, enabling her to find and fix mistakes before resubmission. In this way, the digital ink system captured each student’s attempts at identifying mistakes and how they thought the mistakes should be fixed. This aspect, distinct from other tools such as those described in [32,33] was intentionally built to capture authentic student work. Such student work enables instructors to see how students wanted to solve problems free from interference of hints or a suggested procedure which may be in conflict with the student’s own approach to solving the problem. Furthermore, it helps researchers to explore a student’s thinking process.

5. Results

To assist us with categorizing solutions, the researchers sorted students using the CLASS performance map feature of CLASS according to their performances on five function composition questions from the PCA (see Figure 3). For the following analysis of solutions to quiz questions, we chose cases from a group of students who correctly answered function composition questions from the PCA that involved finding an algebraic formula or using a table or graph, but failed to answer the fifth question that required them to identify function composition in a contextual problem. To illustrate, we share our results from the analysis of two students’ solutions to one quiz question involving the chain rule, and hence function composition, given during week 7 of the semester.

CLASS coding features allow one to create keys and descriptions of student work. This feature was designed to allow researchers to apply open and axial coding procedures as described by Strauss and Corbin [34]. After sorting work according to a CLASS performance map [10], the tool provides researchers with collections of digital work maps representing each student’s attempts at solving the problem. Figure 4 (also in Figure 2) shows the digital work map; this tells the instructor how long a student spent on a problem (purple lines are time spent writing), how many times they erased (pink lines), and how many different answers the student submitted (empty/filled circles). Clicking on any point in the work map shows the student’s work up to that point in time.

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3 There were 40 students at the beginning of the semester, as seen on CLASS performance map of pretest in Figure 3. Six students dropped the course after taking the pre-test and prior to other data collection.

4 The researchers received permission to utilize the PCA instrument in Fall 2009.
Figure 4. Work for Student 2 with annotations.

In addition, an annotation window is opened next to the student’s work (Figure 4). The annotation window provides a place for researchers to assign key codes and comments which are then tied to that specific time marked on the work map. Researchers can instantly supply new key codes and their descriptions or select from a menu of all existing key codes using a text search mechanism. When finished with the annotations, the replay time, associated key codes, and comments are displayed with the researcher’s initials below the student’s work map in chronological order. The annotations are collaborative and editable. By clicking on the replay time, other researchers can instantly jump to the specific replay point and add additional keys or comments.

One problem we chose to analyze was: Find the derivative of the function $f(x) = 12 - 4 \cos(\pi x)$. The problem involved two random parameters (12 and 4) that were different for each student. We selected two students with different answers to this quiz question; however they were on the same branch of the PCA CLASS performance map.

The first student was unsuccessful in his attempts to solve the problem (Figure 5). For this student’s work, the codes of `chain_rule_failure_recognition`, `deriv_const_correct`, `chain_rule_failure_partial_recognition`, `chain_rule_incorrect_implementation`, `d_oversimplification`, `meta_transfer_failure`, and `deriv_trig_correct` were created. These codes are chosen to be descriptive of the errors made by the student. The first code, `chain_rule_failure_recognition` was used because the student did not attempt to use the chain rule at all in his first attempt at the problem. The student attempted to take the derivative of each piece that he came across in the expression. Hence, the student correctly took the derivative of 12 and obtained 0, coded as `deriv_constant_correct`. The student then attempted the derivative of $4 \cos(\pi x)$ by taking the derivative of the constant 4 and getting 0 then proceeding with taking the derivative of $\cos(\pi x)$ as $\sin(\pi x)$. The student was neglecting the
chain rule as well as the fact that they had just found the derivative of the constant in front of cosine to be 0.

Figure 5. Work for Student 1.

Upon submitting his answer and learning that it was incorrect, the student made a second attempt at solving the problem. In the second attempt, we observe that the student now recognizes the need to use the chain rule, as evidenced by his use of a $\pi \cdot 1$ inside the sine function, but does not correctly apply it, as he does not multiply by $\pi$ outside the function. This was initially puzzling to two of the researchers, whose discussion and conjecture with the instructor led to a breakthrough for interpreting the student’s solution process. The missing element was how differentiation had been introduced to the students.

The instructor (co-author and a member of the research team) introduced derivatives as the ratio of two differentials, such as $df$ and $dt$, which represent small changes in $f$ and $t$, respectively. This provides a mechanism which we refer to as the **Marching D** that replaces a single process for derivative rules with a two-step process (“zap with d” and “solve for the appropriate derivative”) incorporating the chain rule, and error-checking mechanisms. For example, when differentiating $f(x) = 12 - 4\cos(\pi x)$, zapping with $d$ produces $df(x) = d(12) - d(4\cos(\pi x))$ or $df(x) = 0 - 4(-\sin(\pi x))d(\pi x)$. The $d(\pi x)$ in the final term indicates the need for an additional differential rule, reducing the expression to $df(x) = -4(-\sin(\pi x))\pi dx$. Notice each term contains a differential, making it possible to solve for the derivative $df/dx = 4\pi \sin(\pi x)$.

The code $d_{oversimplification}$ was created in reference to the **Marching D** as a result of the discussion with the instructor about how he taught the derivative rules. Upon discovering what had been presented in class, it became evident to us that the student was attempting to use the procedure without understanding what he was doing. The student was allowing the $d$ to march through and take derivatives from left to right with no regard for whether there were inside functions. The student did identify the correct derivative of cosine in the final attempt at the problem.

The second student, whose work (Figure 4) we chose to code, successfully solved the problem on the second attempt. This student had $f(x) = 10 - 7\cos(\pi x)$ as his problem. Upon opening the student’s work, we immediately saw very clear evidence of the **Marching**

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5 The term $d(4\cos(\pi x))$ can also be expanded with the product rule, giving $\cos(\pi x) d(4) + 4d(\cos(\pi x))$, but $d(4) = 0$ as four is a constant and does not change. This use of the product rule is evident in second attempt of Student 2 which produced a correct answer.
WeBWorK CLASS: Fostering design experiment research on concept development

As described above. We coded the work using the same codes from the previous coding sessions and created new codes of meta_transfer_partial_success, representation_issue_operations, parentheses_attention, chain_rule_failure_inside, chain_rule_fix_inside, new_strategy_product_rule, and chain_rule_correct_implementation.

This student was successful in identifying his errors and making adjustments, the most significant of which were fixing the chain rule and choosing to use a different strategy. This student chose to view the second part of the problem as a product of two functions. While not necessary to get the right answer, it did allow him to address an error. The student also provided us with something unexpected which led us to create the code representation_issue_operations. The student was able to enter a correct answer, but examination of the written work showed that his notation was lacking. The student wrote $-7 - \sin(\pi x)\ d(\pi x)$ on both attempts, but in typing in his answer he put in a multiplication between the $-7$ and the $-\sin$. This provided evidence that the student was thinking about the operation as multiplication but the written work did not indicate that and communicated a subtraction operation. As a result, student 2 did not accurately communicate how he was thinking about the problem. Such a problem with the associated work is valuable for a teacher to facilitate a classroom discussion about the importance of how one communicates mathematical ideas both in writing and verbally.

After completing the data coding for student 2, the research team examined the codes for student 1 and fine-tuned both the keys and comments. We analyzed other students’ work on similar problems using these existing keys and new codes were added to the list. As a result of this first analysis, we were able to identify misconceptions about the Marching D, which can be used to inform future iterations of the design experiment.

6. Discussion

This brief analysis of two students’ solutions to the same problem demonstrates the use of CLASS to highlight important aspects of students’ learning and discover how students interpret what was presented in lecture. Even though both students had similar scores on the PCA pretest, their attempts and solutions showed different characteristics in how they solved the problem. Analysis illuminates the ways in which both students were trying to imitate the in-class presentation of the Marching D idea. Student 1 did not produce a correct answer, whereas Student 2 was able to fix his mistake leading to a correct answer.

One significant difference between Student 1 and Student 2 is related to students’ abilities to re-assess and re-evaluate their solution process once informed their original answer was incorrect. Student 2 employed a more thorough application of the Marching D idea, and was able to produce a correct answer and solution, while Student 1 failed to recognize the role function composition played in the problem. In fact, this was an issue across students in iteration 1 – told their answer was incorrect, many students were unable to assess their work for mistakes. Many repeated their original solution process, adjusted them in insignificant ways, or changed the form in which they represented their answer without changing the value (incorrectness) of their answer. Iteration 2 was designed to help students analyze their and others’ solutions (both correct and incorrect), discuss possible ways to improve these solutions, and assess different solution strategies. This necessitated introducing CLASS into classroom teaching to create active learning environments through authentic students’ work. Furthermore, incorporation of CLASS in the classroom provided students’ with a coherent learning experience where the same tool is being used both in and out of classroom. There are early indicators from iteration 2 that students are becoming better at reflecting on incorrect solutions and moving beyond mere imitation of procedures.
7. Conclusion
The purpose of this article was to demonstrate how an enhanced online assignment system, WeBWorK CLASS, could be used as a dynamic teaching and research tool in a design experiment project. CLASS assists in capturing, organizing, and analyzing large longitudinal data sets, which are important assets for understanding students’ concept development. The annotation tool allows researchers to quickly find groups of students, with the help of CLASS performance maps, who would be expected to have similar reasoning patterns. Once a group is identified, the annotation tool helps to classify the student’s reasoning skills and problem solving strategies. These classifications help researchers to identify what patterns are emerging in students’ concept development. Similarly, an instructor could use different students’ strategies in class to deepen students’ conceptual understanding through discussions. Our future work will focus more on the use of CLASS in the classroom by analysing data collected in iteration 2 of our design experiment.

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