General Information

The Venue
The venue for the conference is “The Boardwalk” conference centre on Beach Road in Port Elizabeth.

Registration Desk
For general enquiries and assistance, please visit the Registration Desk. The Registration desk is situated on the hotel’s ground level landing next to the entrance to the venues Tsitsikamma D1 and D2.

Disclaimer
Whilst we have endeavoured to ensure all information is accurate, the organisers reserve the right to change any aspect of the programme without prior notice.

Name Badges
Please ensure that you wear your conference name badge at all times during the conference sessions and social functions.

Internet Access
There is free Wi-Fi access at the hotel. The wireless network is Guest@sun and there is no password.

Speaker Support
Speakers should ensure that their presentation is loaded into the system and tested before their session. Technical assistance is available through the Registration Desk.

Social Programme

Welcome Reception
Sunday 22 November, in Tsitsikamma C1 room, 7:00pm till late. Dress code is “casual”.

Social Function
There will be a social function at Havana Blu from 4:00pm till 6:00pm on Monday 23 November. Havana Blu is a short walk from the hotel on Beach Road.

Conference Excursions
Wednesday 25 November is Excursion Day. The following three activities come highly recommended by the local organising committee:

- Addo Elephant Park: Full day tour
- The Penguin boat tour
- The Township tour

Pick up point for these tours will be from the hotel. More information on these tours can be found at www.qualitytouringservices.co.za/delta_conference.php.

Please note that the price of these excursions is not included in the registration fee.

Conference Dinner
The conference dinner will be a traditional South African braai on Thursday 26 November. The venue is The Willows and busses will be available to transport you to and from the hotel. Dress code is “casual”.

---

Elephant Delta

© 2015

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---

Elephant Delta

© 2015
Welcome to Elephant Delta, the Tenth Southern Hemisphere Conference on the Teaching and Learning of Undergraduate Mathematics and Statistics.

We are very pleased indeed to welcome you to Port Elizabeth, the Friendly City and a gateway to the Eastern Cape. Our programme, within the bounds of the topic of the conference, is varied and promises much opportunity for learning and for networking with peers across the Southern Hemisphere, with our Northern Hemisphere colleagues joining us under the African sun. The publications for this conference include the special issue 46(7) of the International Journal of Mathematical Education in Science and Technology, the Proceedings, the Communications and the Programme, all drawn up and edited by the Elephant Delta committee. The iJMEST issue and the Proceedings were double blind peer reviewed by at least two reviewers per paper.

We have worked hard to ensure that this conference is worthy of the name of “Delta”, living up to the high standards of the conferences which have preceded this one. We hope that you will enjoy the conference, the city and your experiences in the Eastern Cape.

In pursuit of excellence in teaching and learning,

**Tracy Craig, on behalf of the organising committee**

---

**The committee is**

Tracy Craig *(University of Cape Town)* - chair
David Holgate *(University of the Western Cape)* – assistant chair
Rénette Blignaut *(University of the Western Cape)*
Anita Campbell *(University of Cape Town)*
Johann Engelbrecht *(University of Pretoria)*
Rita Kizito *(UWC/Nelson Mandela Metropolitan University)*
Batseba Mofolo-Mbokane *(University of Pretoria)*
Pragashni Padayachee *(Nelson Mandela Metropolitan University)*
Maritz Snyders *(Nelson Mandela Metropolitan University)*
Sarie Snyders *(Nelson Mandela Metropolitan University/independent)*
Stuart Torr *(University of Cape Town)*
Esmé Voges *(Tshwane University of Technology)*
John Webb *(University of Cape Town)*
Abbreviations

[A] Abstracts only

[P] Full paper in the conference proceedings

[IJMEST] Paper published in the International Journal of Mathematical Education in Science and Technology

SUNDAY 22 November

| 16:00 – 18:00 | REGISTRATION  
|               | Tsitsikamma C1 |
| 19:00 – late  | WELCOME RECEPTION  
|               | Tsitsikamma C1 |
## MONDAY 23 November

### 08:45 – 09:00
WELCOME TO ELEPHANT DELTA

### 09:00 – 10:00
**KEYNOTE:** Prof Tim Dunne: *The Rasch model for test outcomes and related item requirements*

### 10:00 – 10:30
MORNING TEA

### 10:30 – 12:30
**MONDAY: MULTIPLE STREAM SESSION 1**

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<thead>
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<th>Theme: Assessment Tsitsikamma C1</th>
<th>Theme: Teaching and learning practices Tsitsikamma D1</th>
<th>Theme: Extended programs Tsitsikamma D2</th>
</tr>
</thead>
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LUNCH
### MONDAY: MULTIPLE STREAM SESSION 2

<table>
<thead>
<tr>
<th>Time</th>
<th>Theme: Teaching and learning practices</th>
<th>Theme: Teaching and learning practices</th>
<th>Theme: Teaching and Learning practices</th>
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</thead>
</table>

<table>
<thead>
<tr>
<th>Time</th>
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<tr>
<td>15:30 – 16:00</td>
<td>AFTERNOON TEA</td>
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<tr>
<td>16:00 – 18:00</td>
<td>SOCIAL FUNCTION AT HAVANA BLU</td>
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<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
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</thead>
<tbody>
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<td><strong>KEYNOTE:</strong> Dr Deborah King: <em>Think Big! A local initiative that became a National Network</em></td>
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<tr>
<td>10:00 – 10:30</td>
<td><strong>MORNING TEA</strong></td>
</tr>
<tr>
<td>10:30 – 12:30</td>
<td><strong>TUESDAY: MULTIPLE STREAM SESSION 3</strong></td>
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<td></td>
<td><strong>Theme: Professional development</strong></td>
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<tr>
<td></td>
<td>Tsitsikamma C1</td>
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<tr>
<td>10:30 – 10:55</td>
<td>Theoretical and Empirical Approaches to Abstraction in Mathematics Education [A] Stuart Torr</td>
</tr>
<tr>
<td>11:00 – 11:25</td>
<td>Placing undergraduate mathematics assessment on the national Higher Education agenda [A] Deborah King, Cristina Varsavsky</td>
</tr>
<tr>
<td></td>
<td><strong>Theme: Maths Educators</strong></td>
</tr>
<tr>
<td></td>
<td>Tsitsikamma D1</td>
</tr>
<tr>
<td>11:00 – 11:25</td>
<td>Advanced mathematics curriculum for students preparing to teach middle school mathematics [A] Gary Harris</td>
</tr>
<tr>
<td>11:30 – 11:55</td>
<td>Opening Real Science: Engaging Tomorrow’s Science and Mathematics Teachers [A] Leigh Wood</td>
</tr>
<tr>
<td>12:00 – 12:25</td>
<td>Investigating how past experiences in mathematics have influenced pre-service primary teachers [A] Dilshara Hill</td>
</tr>
<tr>
<td></td>
<td><strong>Theme: Technology</strong></td>
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<tr>
<td></td>
<td>Tsitsikamma D2</td>
</tr>
<tr>
<td>Time</td>
<td>Event</td>
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<tr>
<td>12:30 – 14:00</td>
<td>LUNCH</td>
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<tr>
<td>14:00 – 15:30</td>
<td>TUESDAY: MULTIPLE STREAM SESSION 4</td>
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<td></td>
<td><strong>Theme: Teaching and Learning practices</strong></td>
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<td></td>
<td><strong>Tsitsikamma C1</strong></td>
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<tr>
<td>14:00 – 14:25</td>
<td>An Analysis of the Reasoning Abilities of Students in the Transition Period from Secondary to Tertiary Mathematics [P] Trudie Benadé and Sonica Froneman</td>
</tr>
<tr>
<td></td>
<td><strong>Theme: Maths Educators</strong></td>
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<td></td>
<td><strong>Tsitsikamma D1</strong></td>
</tr>
<tr>
<td>14:00 – 14:25</td>
<td>Numeracy and Mathematical Skills of Pre-service Primary Teachers [A] Therese Wilson, Bronwyn Ewing, Daniel Crane, Charisse Farr</td>
</tr>
<tr>
<td></td>
<td><strong>Theme: Technology</strong></td>
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<tr>
<td></td>
<td><strong>Tsitsikamma D2</strong></td>
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<tr>
<td>14:00 – 14:25</td>
<td>Western Cape Maths4 Stats training 2012 to 2014 – valuable or not? [A] Renette J Blignaut, Abduraghiem Latief, Ronell Jacobus, Retha Luus, Morne Lamont, Rechelle Jacobs</td>
</tr>
<tr>
<td>14:30 – 14:55</td>
<td>The impact of assumed knowledge entry standards on undergraduate mathematics teaching in Australia [IJMEST] Dr Deborah King and Ms Joann Cattlin</td>
</tr>
<tr>
<td>14:30 – 14:55</td>
<td>Zimbabwean pre-service teachers’ responses to matrix algebra assessment items [P] Cathrin Kazunga and Sarah Bansilal</td>
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<tr>
<td>15:00 – 15:25</td>
<td>Using clickers in the mathematics classroom: novelty or necessity [A] Karin Bothma</td>
</tr>
<tr>
<td>15:00 – 15:25</td>
<td>Great Expectations: expectations and attitudes of first-year mathematics students [A] Leanne Rylands and Don Shearman</td>
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<tr>
<td>15:30 – 16:00</td>
<td>AFTERNOON TEA</td>
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<tr>
<td>16:00 – 17:30</td>
<td>PLENARY SESSION</td>
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<tr>
<td></td>
<td><strong>Theme: Collaboration</strong></td>
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<td></td>
<td><strong>Tsitsikamma C1</strong></td>
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**WEDNESDAY 25 November**

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<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
<th>Description</th>
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<tbody>
<tr>
<td>09:00 – 10:10</td>
<td>Tsitsikamma C1</td>
<td><strong>KEYNOTE:</strong> Prof Maxine Pfannkuch, the Judy Patterson speaker: <em>Visualizing chance in introductory probability</em></td>
</tr>
<tr>
<td>10:10 – 10:30</td>
<td>MORNING TEA</td>
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<tr>
<td>10:30 – 12:30</td>
<td><strong>THURSDAY: MULTIPLE STREAM SESSION 5</strong></td>
<td><strong>Theme: Statistics</strong>&lt;br&gt; Tsitsikamma C1  &lt;br&gt;&lt;br&gt; <strong>Theme: Curriculum design and delivery in the 21st century</strong>&lt;br&gt; Tsitsikamma D1  &lt;br&gt;&lt;br&gt; <strong>Theme: Teaching and Learning practices</strong>&lt;br&gt; Tsitsikamma D2</td>
</tr>
<tr>
<td>11:00 – 11:25</td>
<td>Influence of matric mathematics versus mathematical literacy in the success rate of an introduction course in statistics at TUT [A]&lt;br&gt;<em>SA Mouton &amp; I Louw</em></td>
<td>Learning from experience: The realities of developing mathematics courses for an online engineering program [IJMEST]&lt;br&gt;<em>Diana Quinn, Amie Albrecht, Brian Webby</em></td>
</tr>
<tr>
<td>Time</td>
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<tr>
<td>12:30 – 14:00</td>
<td><strong>LUNCH</strong></td>
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<td>14:00 – 15:00</td>
<td><strong>KEYNOTE:</strong> David Wagner: <em>Mathematics, Culture, and the Responsibilities of Mathematics Educators</em></td>
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<tr>
<td>15:00 – 15:30</td>
<td><strong>PLENARY SESSION</strong></td>
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<td>Delta 2017</td>
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<td><em>The Brazilian team</em></td>
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<td>15:30 – 16:00</td>
<td><strong>AFTERNOON TEA</strong></td>
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<td>16:00 – 16:25</td>
<td><strong>PLENARY SESSION</strong></td>
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<td></td>
<td>Achieving Sustained Positive Outcomes in Maths and Stats Education: An Efficacy Study [A]</td>
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<td><em>Pearson team (Beverley Wharton-Hood /Seshni Mala)</em></td>
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<td>16:30 – 17:30</td>
<td><strong>THURSDAY: MULTIPLE STREAM SESSION 6</strong></td>
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<td></td>
<td><strong>Theme:</strong> Miscellaneous</td>
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<td></td>
<td>Tsitsikamma C1</td>
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<tr>
<td>16:30 – 16:55</td>
<td>Conceptual or procedural mathematics for engineering students – views of two qualified engineers from two countries [IJMEST] Christer Bergsten; Johann Engelbrecht, and Owe Kågesten</td>
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<td></td>
<td><strong>Theme:</strong> Teaching and learning practices Tsitsikamma D1</td>
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<td>Mathematics pre-knowledge for entering Economic and Management Sciences [A]</td>
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<td></td>
<td><em>Mardi Jankowitz, Ilsa Basson, Adele Immelman, Machteld Strydom</em></td>
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<td></td>
<td><strong>Theme:</strong> Teaching and learning</td>
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<tr>
<td></td>
<td>Tsitsikamma D2</td>
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<td></td>
<td>Multicultural lecturing: some challenges [P]</td>
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<td><em>Jyoti Jhagroo</em></td>
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<tr>
<td>17:00 – 17:30</td>
<td>Optimization in first semester calculus: A look at a classic problem [IJMEST]</td>
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<td><em>Renee LaRue and Nicole Engelke Infante</em></td>
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<td><strong>Threshold concepts in finance: student perspectives [IJMEST]</strong></td>
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<td><em>Susan Hoadley, Tim Kyng, Leonie Tickle, Leigh N. Wood</em></td>
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<td><strong>An Investigation into the Effect of Mathematics Self-Efficacy and Mathematics Anxiety on Mathematics Performance [A]</strong></td>
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<td><em>SJ Wagner-Welsh, M Watson</em></td>
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<tr>
<td>18:30 – late</td>
<td><strong>BRAAI AT THE WILLOWS</strong></td>
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<table>
<thead>
<tr>
<th>Time</th>
<th>Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 – 10:00</td>
<td><strong>KEYNOTE:</strong> João Frederico da Costa Azevedo Meyer: <em>Mathematical Disciplines for Undergraduate (and Graduate) Mathematics and Statistics: Challenges for Cooperating and Operating with Social Needs</em></td>
</tr>
<tr>
<td>10:00 – 10:30</td>
<td>MORNING TEA</td>
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<tr>
<td>10:30 – 12:30</td>
<td><strong>FRIDAY: MULTIPLE STREAM SESSION 7</strong></td>
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<td></td>
<td><strong>Theme: Statistics</strong></td>
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<td></td>
<td>Tsitsikamma C1</td>
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<td></td>
<td><em>Greg Oates</em></td>
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<td><em>Richard Wilson</em></td>
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<tr>
<td>11:30 – 11:55</td>
<td>First level statistics students’ performance in a large classroom environment under the magnifying glass [A]</td>
</tr>
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<td></td>
<td><em>Fransonet Reyneke</em></td>
</tr>
<tr>
<td>12:00 – 14:00</td>
<td>CLOSING CEREMONY AND LUNCH</td>
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Abstracts of Oral Presentations

MONDAY: Keynote speaker

Prof Tim Dunne, University of Cape Town, South Africa

Tim Dunne is an Emeritus Professor of Statistical Sciences at the University of Cape Town. He has a long-standing enthusiasm for education and its particular challenges in offering mathematics and statistics in both privileged and unprivileged environments. His perspective on lecture and classroom is influenced by advances in assessment theory and design, and by the impact of Rasch measurement theory for teacher diagnoses of current teaching and learnings needs.

The Rasch model for test outcomes and related item requirements

Summary on page 43.

MONDAY: Multiple Stream Session 1

Tsitsikamma C1: Assessment

Theoretical perspectives on peer learning and how they can help in first year mathematics

Anita Campbell

University of Cape Town, South Africa

Active learning involving peer learning has been used with success in undergraduate mathematics. However, simply organising peers in groups and "hoping for the best" tends to yield peer learning with infrequent and simple questioning, missed opportunities to correct errors, and inappropriate positive feedback. Meta-reviews of peer learning recommend coordinating multiple theoretical lenses to gain a fuller understanding about peer learning. The major learning theories underpinning peer learning and have identified as constructivist theories including cognition theories such as cognitive functioning, sociocultural theories and the idea of communities and landscapes of practice. This presentation will summarise these theories and explain how they contribute to better teaching and learning in first year mathematics.
Assessments: An Open and Closed Case

R. Nazim Khan

University of Western Australia, Australia

Article published in IJMEST, Vol. 46, No. 7.
http://dx.doi.org/10.1080/0020739X.2015.1076534

Open book assessment is not a new idea, but it does not seem to have gained ground in higher education. In particular, not much literature is available on open book examinations in mathematics and statistics in higher education. The objective of this paper is to investigate the appropriateness of open book assessments in a first-year business statistics course. Data over two semesters of open book assessments provided some interesting results when compared with the closed book assessment regime in the following semester. The relevance of the results is discussed and compared with findings from the literature. The implications of insights gained for further practice in the assessment of mathematics and statistics is also discussed.

Are students actually learning what we intend to teach them? An analysis of student responses to first-year calculus examination questions

Justin Munyakazi, Rita Kizito and Jacques Elengemoke

University of the Western Cape, South Africa

The consistently low pass rate observed in the last couple of years (21% in 2011, 26% in 2012, 29% in 2013 and 31% in 2014), in a first-year Mathematics module at a South African University raises the question of adequacy of assessments to which students are subjected. Student performance is particularly low in the Calculus sections of the examinations. In seeking to understand the reasons behind the low performance in calculus, this paper describes the analysis of student responses to a selection of first-year calculus examination questions given at the end of the academic year. The sources of data were responses belonging to a sample of 36 students systematically selected from a group of 138 students. The MATH (Mathematical Assessment Task Hierarchy) taxonomy was used to distinguish between the levels of difficulty for each question. The constant comparative method was then used to identify salient similarities and differences in the student’ answers. The findings from the study highlight the types of questions students find difficult and the links between questions and the types of questions that students find difficult. More importantly, it raises the pedagogical question of whether students are actually learning what we intend to teach them.

Experimenting with assessment

Rachel Passmore

University of Auckland, New Zealand

Inspired by David Holgate’s presentation at the last DELTA conference in Kiama, I decided to experiment with different types of assessments for my own students. I teach a Foundation Mathematics course at the University of Auckland. This course is part of
a Tertiary Foundation Certificate which offers students a second chance at gaining a University Entrance qualification. As well as David’s example of assessment by video I also considered the National Council of Teachers of Mathematics’ (NCTM) suggestions for alternative forms of assessment.

My first experiment involved assessment by video. There were many challenges involved with this sort of assessment both for me and for the students. Aside from the obvious technical issues, choice of topic, a marking scheme and evaluation all required consideration. Clips from videos will be shown from each of the two years that I have used this assessment. The creativity of the students is amazing. Student test results for the topic of the video were compared pre-making of the video and post-making of the video. The results surpassed all my expectations.

I have also experimented with an assessment based on one of Dan Meyer’s three-act scenarios, which essentially requires the students to formulate their own mathematical questions after viewing a short video clip. Students were very uncomfortable with formulating their own questions, so in the second year of this assessment a slight refinement produced more successful results.

A third experiment with assessment pursued my theme of encouraging creativity in Mathematics. This third experiment involved students using the on-line free graphing software, DESMOS, to replicate and design company logos.

Tsitsikamma D1: Teaching and learning practices

Interventions to improve teaching and learning in first year mathematics courses

Johann Engelbrecht and Ansie Harding

University of Pretoria, South Africa

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In keeping with the national mandate of increasing graduates in the sciences in South Africa, a concerted effort in improving the first year experience becomes imperative. First year mathematics courses commonly provide the base knowledge necessary for progression in different degree programmes at university. Success in mathematics courses influences throughputs, retention and graduation rates of various degree programmes. Due to the highly complex and integrated nature of issues pertaining to improving teaching and learning in these courses, a multi-dimensional approach was conceptualized and implemented at the University of Pretoria. This paper reports on the development of a coherent framework, and the process and strategy for improving student success through a number of teaching and learning interventions in the first year mathematics courses, addressing the different dimensions of the framework. The process embarked upon resulted in a coherent, resource-focused approach with a replicable model for similar contexts.
Looking into the Private World of the Private Tuition Industry
Anne D'Arcy-Warmington

Statistica, Perth, Australia

Education is often a top priority for many parents initially, as their child approaches school age and again, as high stake examinations are close on the horizon. As the child progresses through the education system, parents can supplement formal education in different ways, such as requesting extra homework from the current education institution; purchasing publications that target examination style questions; internet search for learning resources or seek private tuition. Private tuition (also known as coaching, shadow education, and parallel education) may be conducted in person as one-to-one, small group, or in larger groups in a classroom style setting. There are on-line options available such as self-guided learning programs and real-time interaction with tutors. As the uptake of employing private tutors is increasing globally, the industry has become one of the fastest-growing sectors of the education field. It is surprising that in many countries, there is very little regulation and monitoring of this industry given the potential income that can be generated. This qualitative paper will seek to highlight the drivers behind the demand for private tuition and explore regulations and government interventions in the private tuition industry. This paper will define private tuition as any academic assistance rendered outside the education institution for a monetary fee.

The influence of student engagement on the performance of first-year mathematics students: A Case Study
Elsabe Weyer

North West University, South Africa

Student engagement, in general, has been researched extensively by a number of researchers (Kuh et al., 2006; Gasiewski et al., 2012). Zepke, Leach and Butler (2010) state that the following factors influence students’ success in general: employment, family, social, cultural and personal factors and more specifically, Briggs (2004) argues that student engagement influences students’ success in mathematics.

In this study several facets of student engagement were investigated. The National Survey of Student Engagement (NSSE) divides student engagement into five dimensions: level of academic challenge, active and collaborative learning, supportive campus environment, enriching educational experiences and student-faculty interaction. The researcher explored specifically the influence of student engagement on performance of first-year mathematics students in their first semester according the above mentioned five dimensions.

A mixed method approach of qualitative and quantitative measurements was used.

• A pilot study was done in 2014 by administering the National Survey of Student Engagement (NSSE) to ascertain whether the adjusted questions, in the survey, specifically for mathematics students, were correctly formulated.

• Quantitative: The National Survey of Student Engagement (NSSE) was administered at the end of the first semester of 2015 to the first-year Mathematics (WISN111) students at the Potchefstroom Campus of the North-West University,
approximately 300 students.

- Qualitative: After the analysis of the data collected in the survey, interviews were done with a maximum of six students. A purposive sample of participants included two students which were highly successful in WISN111, two students with average marks for WISN111 and two students which failed WISN111.

**Cause-effect Analysis: Improvement of a 1st year Engineering Students’ Calculus Teaching Model**

**Quay van der Hoff, Ansie Harding**

**University of Pretoria, South Africa**

This study focuses on the mathematics department at a South African university and in particular on teaching of calculus to first year engineering students. The paper reports on a cause-effect analysis, often used for business improvement. A reality tree was constructed through critical thinking from which the core problem was identified. The cause-effect analysis indicates that there are many factors that impact on secondary school teaching of mathematics, factors that the tertiary sector has no control over. This analysis also indicated the undesirable issues that are at the root of impeding success in the calculus module. Most important is that students are not encouraged to become independent thinkers from an early age. This triggers problems in follow up courses where students are expected to have learned to deal with the work load and understanding of certain concepts. A new model was designed to lessen the impact of these undesirable issues.

**Tsitsikamma D2: Extended programs**

**Using history to help bridging learners see embedded mathematics**

**Phil Kane**

**University of Auckland, New Zealand**

A major claim for learning mathematics is to support growth in technology (Ernest, 1998; Schliemann, 2004). The technology of warfare however, is an ‘elephant in the room’ in that although warfare has benefitted from mathematics and other scientific ideas, it is perhaps not discussed as fondly. With it being over one hundred years since the start of World War One (WW1), an opportunity arose to use an episode involving New Zealanders in that conflict, alongside some relevant mathematical ideas. Although we seek contexts from science and commerce to help show where mathematics is embedded, many of our Tertiary Foundation Certificate (TFC) learners study history in preparation for entry to a BA, and are no longer involved in scientific subjects. So early in the module on functions, I introduced the class to catapult motion to help lead them towards parabolic curves. The creeping development of artillery technology was also discussed with the class, and eventually led us to WW1 where students were asked about what they knew of Passchendaele. No one recognised the place, and only one had heard of the word. However, when ‘Gallipoli’ was mentioned, most of the class
recognized New Zealand’s involvement one hundred years ago as part of a wider Anzac contingent. With much less publicity, the October 1917 battle near Passchendaele produced our country’s heaviest defeat in the war, in what is now considered an extraordinarily ill-prepared battle (Harper, 2004). The hope was that these learners might also understand how some of the parabolic ideas and the leadership at Passchendaele failed our soldiers so badly in 1917.

Developing Academic Maturity of Students in an extended programme in Mathematics
Bridgette Makhosazana Yani

*University of Pretoria, South Africa*

This paper focuses on the students who are registered under the University of Pretoria’s Academic development programmes, named the Four-year programmes (FYP). The programmes were first introduced in 2008 as a gateway for students who were underprepared but are willing to work diligently and continue their studies into the science mainstream programmes. The research focuses on measuring the growth in academic maturity in the students who are entered for this programme. Academic maturity is subdivided into two components namely non-subject based maturity and subject based maturity (mathematical maturity). For measuring non-subject based academic maturity, students participate in a survey at the beginning of the year during the university’s orientation week and after the first six months at university. The survey done is called the Student Academic Readiness Survey (STARS), taken at the beginning of the year and after the first semester respectively. For the subject-based component students write two mathematics tests testing their mathematical skills a pre- and post-test before and after six months of formal teaching. The results of the surveys are compared to measure to what extent students’ views change and the mathematics pre-test will be compared to the post-test to measure the extent of improvement in students’ confidence and mathematical skills.

Unisa Science Foundation Provision: From the Augmented to the Extended Model for teaching Mathematics in an Open Distance Learning environment
Belinda Huntley

*University of South Africa, South Africa*

Foundation Provision is a national academic support programme in most South African Universities and Institutes of Higher Learning which is funded by the Department of Higher Education and Training (DHET) through a foundation grant. The main purpose of Foundation Provision is to improve the academic performance of those undergraduate students who comply with the minimum requirements to enrol for a particular university qualification, but who are at risk of dropping out or failing due to their poor educational backgrounds. The goal of Science Foundation Provision (SFP) at the University of South Africa (Unisa) is equity of both access and success.
In 2015, an institutional review of SFP at Unisa proposed the move from the Augmented Model to the Extended Model, but retaining the mix of extended (theory) and regular (mostly practical) modules in the 1st year. In the Department of Mathematical Sciences at Unisa, eight first-year SFP Mathematics modules are taught. An online blended approach of teaching and learning within the context of the Extended Model is used in the delivery of these modules. The various methods of content delivery, assessment and pedagogy in an online environment will be discussed with particular reference to the SFP Extended Model. In planning for the Extended Model, SFP has been redesigned to include a number of interventions and formative assessment opportunities to improve student activity and success. Unisa’s Science Foundation Provisions have been aligned with Unisa’s best Open Distance Learning (ODL) practices and technologies in order to improve the quality, and to broaden the modes of learner support.

Reforming a preparatory year maths module to improve attainment and engagement

Dr Hannah Bartholomew

Sheffield Hallam University, United Kingdom

Foundation Mathematics 1 is a core module in a preparatory year at Sheffield Hallam University. It attracts 60 - 80 students per year, and most students completing this course go on to an engineering or mathematics degree at Hallam. The module covers a range of secondary school mathematics topics including fractions, decimals, working with algebraic expressions and equations, linear and quadratic functions, and right angled triangle trigonometry (most students go on to meet calculus, exponential and log functions and further trigonometry in FM2).

As module leader I have become increasingly frustrated with aspects of the way it is structured and assessed. In particular:

- The assessment was insufficiently varied. The questions students encountered were similar in style, and many of the skills that we might hope to develop in students were not assessed.
- I struggled to adequately engage the full range of students taking the module, and the highly content driven approach made it very hard to pitch my teaching at a level that suited everyone.

During the 2014/15 academic year I made significant changes to the delivery and assessment of the module, in the hope of ameliorating some of these problems. In this talk I will describe the changes that I have made to the assessment of this module, explaining the rationale behind them, and discussing the parallel changes that I made to the way that I deliver the module. I will then draw some preliminary conclusions about the impact these have had on students' engagement and performance in the module.
Tsitsikamma C1: Teaching and learning practices

Developing problem-solving skills through oral assessment
Heather Lonsdale

*Curtin University, Australia*

Problem-solving skills are a key graduate attribute for mathematics, and should be an important focus of the curriculum and assessment. To develop these skills, with formative feedback on the process of problem-solving, it can help to encourage students to talk through the process: both among themselves and with a tutor. These valuable conversations happen readily in a “flipped tutorial” setting, run in the collaborative board-work style sometimes referred to as the La Trobe method (Seaton, 2014). To place further value on these conversations, and to constructively align the assessment with the learning activities, oral assessment was introduced into a first-year calculus unit at Curtin University. This involved students discussing problems in small groups, with a tutor circulating to ask the students to explain the problem-solving process. Students were assessed on their explanation of what methods were available, how to apply them, and why each method would or would not work. The assessment occurred three times throughout the semester, and video data was collected to analyse the students’ improvement over this time. This presentation will discuss the implementation of this oral assessment, student perception, and its role of “assessment as learning”.

Ethics approval: Curtin University Human Research Ethics Committee (HREC) has approved this study (HREC number RDSE-43-15).

Skills involved in teaching large Groups of undergraduate Students

Ansie Harding, Johann Engelbrecht

*University of Pretoria, South Africa*

The study is set at a large research intensive university in South Africa. The teaching model in mathematics for entry level students is that of large group teaching, with up to five hundred students per group. Principles for the success of large group teaching in mathematics are identified by the teachers involved. These principles are classified hierarchical into two broad categories, the first concerning organisational principles and the second involving social principles based on the human element. Each category is subdivided into three sub-categories. The study shows, as expected, that organisational and administrative fluency are critical principles for both students and teachers. The importance of suitable and well-equipped venues as well as skilful use of technology is perceived as essential. What does emerge, and should be taken notice of, is the importance of “soft” skills. Such skills include knowledge of large group thinking, the ability to deploy strategies to build a group identity, also being able to devise activities to build coherence and for making the individual feel recognised. The recommendation
is for these skills to be developed for cultivating an environment within which large group learning is optimised.

Are we leaving mathematics students starved and bored in class?

Harry Wiggins

*University of Pretoria, South Africa*

This paper spotlights student enrichment in mathematics at university level. We consider the pedagogical problem on how to teach problem solving skills to avoid the trap of anti-didactical inversion, that is only teaching the final polished mathematical approach without showing its discovery or evolution over time and to find suitable ways to challenge academically strong students in the classroom. We show how student enrichment is a way of nurturing academically stronger students to keep them enthusiastic and to help them discover their own potential. Student enrichment is therefore a vehicle to further stimulate interest in mathematics by creating opportunities for students to explore, be creative and curious. The paper offers an enrichment model using inquiry-based learning for keen and interested students at university level to sharpen their problem solving skills and to show students how to tap into their own potential by allowing them to explore mathematics in a guided fashion. The model is based on enhanced undergraduate content stemming from a problem in complex numbers on how to visualize the roots of a function which lead to the theory of sibling curves. Using the principles of inquiry-based learning a group of first year students were exposed through five guided activities to explore polynomials, complex numbers and sibling curves by themselves. The paper reports on the experiences, discoveries and successes that the programme had on a group of undergraduate students as well as on and the designer of the enrichment programme.

Tsitsikamma D1: Teaching and learning practices

Moving away from traditional course delivery

Mike Robinson

*Sheffield Hallam University, United Kingdom*

In this presentation we shall discuss significant changes made to the delivery of one of our modules, Dynamical Systems and Fourier Analysis. This was previously delivered in a lecture plus (PC-enabled) tutorial format. Much has been written about the failings of a traditional lecture, and in general we make efforts to ensure that lectures provide a more active learning environment than an old-style 50 minute exposition. However, in this module, the length of many of the calculations, and the extensive use of technology, means there are limited opportunities for meaningful interactivity in a lecture. This is exacerbated by some students’ tendency to think that, whilst lecture attendance is important, tutorial attendance is less so. From this academic year, the delivery has been substantially redesigned to address these issues, taking advantage of both new teaching spaces, and new technologies. Classes now consist of a single 2-hour session (with a group of 50 students) in a classroom designed to encourage group working, with laptops available. Most (but not all) new material is presented in a series of video
mini-lectures or printed notes, which students are expected to engage with before the class. Class activities consist of a variety of activities designed to enable students to explore the material, often working in groups, with some additional material presented. The changes are designed to promote engagement, support peer learning, and deepen the students' understanding. In the presentation we will discuss the challenges faced, the advantages of this approach, and the student reaction to it.

Does the chalkboard still hold its own against modern technology in teaching mathematics? A case study

Anneli Billman

University of Pretoria, South Africa

This case study explores the integration of technology into teaching at a mathematics department at a large South African University. The study shows that use of chalkboards has decreased significantly over the past ten years, while the use of modern technologies has increased accordingly. Teaching of large groups has necessitated the use of technology in the classroom. However, the study also shows that half of the staff members feel that chalkboards are still more suitable than technology for teaching mathematics. This finding supports the idea of a strong subject culture. Age does not emerge as a moderating factor for preference of either technology or the chalkboard, but gender, academic qualifications and teaching qualifications seems to be moderating factors. Subject culture is strongly rooted under the male members of staff, while female staff members feel more positive towards the use of technology in teaching. Results suggest a higher academic qualification or an absence of a teaching qualification indicate a stronger preference for the chalkboard. Despite the existence of a strong subject culture, a shift in attitude towards technology use in teaching is noticed and there is a definite trend of moving towards using new technologies.

The challenge of applying trigonometry to real life situations at first year level

Jeanetta du Preez

University of Pretoria, South Africa

Professional Orientation is a project-based skills focused course for first year engineering students and is one of the compulsory modules in the Engineering Augmented Degree programme (ENGAGE) at the University of Pretoria. The two consecutive modules for Professional Orientation entails four projects and each involves mathematics. One of the projects is the Logo project which requires pre-knowledge of trigonometry and geometry. In order to revise trigonometry identities learnt at school, students do a trigonometry assignment and subsequently write a test. The results of the test show that students can easily solve a triangle using trigonometry. The transfer of the same knowledge to apply it in real life situations, is however not so successful. What this talk will address is the results of a semester test where students had to solve an isosceles triangle by extracting the data from the description given. A large number of students struggled to extract the necessary dimensions to draw a pen and paper diagram (planning diagram) to aid in solving the triangle. The planning
diagram is essential to first plan the logical flow of the procedure for the specific isosceles triangle and then use it to write a basic procedure to draw the triangle using Terrapin Logo software. Finally, the students had to draw a sector of a prototype SKA dish comprising three nested isosceles triangles of a certain ratio.

Tsitsikamma D2: Teaching and learning practices

Simple rule, hidden meaning: the scalar product in engineering mathematics

Tracy S. Craig, Trevor J. Cloete

University of Cape Town, South Africa

Engineering is a highly mathematical field of study with different university courses requiring proficiency at different types of mathematics. Engineering dynamics requires the skilful use of vectors in various ways and proficiency at vector arithmetic, algebra and geometry is of vital importance to incoming students. This paper reports on findings from the administering of a vector proficiency assessment instrument across two semesters of dynamics students, which suggest that problems requiring use of the scalar product embedded within a context are of the highest difficulty level. We argue that the geometric role of the scalar product is weakly understood by the majority of students, leading to poor performance at any problem requiring more than a basic calculation. We suggest that lecturers of engineering mathematics foreground the geometric role and that lecturers of engineering courses be aware of the level of challenge manifest in these problems.

Engineering students’ use of intuition and visualization in the mathematical problem solving

Chih Hsien Huang

Ming Chi University of Technology, Taiwan

Although deciding on the truth value of mathematical statements is an important part of the proving process, students are rarely engaged in making such decisions. Thus, little is known about the ways in which students’ use mathematical reasoning to evaluate mathematical statements. In this study, task-based interviews were conducted with engineering students in which they were asked to determine the truth value of mathematical statements. Students’ reasoning on the tasks will be classified and then further categorized according to the findings of current research, with new categories added as needed. This study should contribute to our understanding of the ways in which students’ reason when dealing with uncertainty in the problem solving process. The findings suggest that the factors the participant failed to solves problems include: mathematical intuition hindered the constructing of counterexamples, too much focus on symbol representations. Additionally, this study may suggest ways in which educators can assist students in navigating the often difficult process of refuting mathematical statements.
Transfer of mathematical learning to science: first year university students’ and academics’ perspectives

Yoshitaka Nakakoji, Rachel Wilson and Leon Poladian

The University of Sydney, Australia

First year students’ poor preparedness of mathematics is a critical issue concerning academics in STEM disciplines. It is important to develop students’ ability to apply mathematical skills and knowledge to their disciplinary contexts for their success in their learning. However, despite of its importance, there was a scant of literature about this issue in the context of higher education. Our naturalistic inquiry investigates this issue from both students’ and academics’ perspectives. Two research strategies are employed for each perspective. Firstly, student case studies (n=10) examine the process of transfer, using a think-aloud method. In addition, individual interviews highlight the difficulties in relation to transfer and how students perceive the relationships between mathematics and science. Secondly, a Delphi study (n=8) focuses on academic perspectives and outlines mathematics and science academics’ views on the relationship between these disciplines and issues surrounding transfer of learning between them.
TUESDAY: Keynote speaker
Dr Deborah King, University of Melbourne, Australia

Dr Deborah King is Coordinator of Learning and Teaching Innovation in the School of Mathematics and Statistics at The University of Melbourne. During her seven-year tenure as Director of First Year Studies, she developed a keen interest in mathematics education, particularly in issues relating to students transition from secondary school to tertiary study and how these affect the professional practice of academics who teach first-year tertiary mathematics. Recently she has been involved in a national project to investigate assessment practices in tertiary mathematics, and has also established the First Year in Maths network of tertiary mathematics educators, which aims to drive change in the teaching of tertiary mathematics.

Think Big! A local initiative that became a National Network
Summary on page 46.

TUESDAY: Multiple Stream Session 3

Tsitsikamma C1: Professional development

Theoretical and Empirical Approaches to Abstraction in Mathematics Education

Stuart Torr
University of Cape Town, South Africa

Understanding abstract mathematical concepts is an important part of mathematical proficiency and it is something that many students struggle to do. As a result, understanding the nature of mathematical abstraction and how to foster this in students is an active area of research in the mathematics education community and there are several related, but distinct approaches to the topic. The two most prominent of these are the empiricist and dialectic accounts. Empiricist accounts, based on the work of Piaget and his notion of reflective abstraction, suggest a linear development, from concrete, lower level conceptions to higher level, abstract ones. The dialectic account on the other hand suggests that level of abstraction is contextually dependant on a subject’s personal history and previously acquired concepts. In addition, the dialectic account points out that as we learn about a concept and the more connections we make to other concepts it becomes more structured and concrete. Until recently, most of the work in this area has been theoretical due to the complexity of the subject and the difficulty of observing the process as opposed the results of successful abstraction. Increasingly, empirical work is being done to ground the theory and to develop more
effective instructional techniques. I argue that Construal level theory can be a tool for the empirical study of abstraction in mathematics and that the theory be used to inform educational practice in a way that fosters the learning of mathematical abstractions.

Placing undergraduate mathematics assessment on the national Higher Education agenda

Deborah King\textsuperscript{1}, Cristina Varsavsky\textsuperscript{2}

\textsuperscript{1}The University of Melbourne, Australia
\textsuperscript{2}Monash University, Australia

The paucity in scholarly work on undergraduate mathematics assessment is an indication that assessment and grading practices in mathematics departments have remained relatively unchanged for many decades. In the current higher education environment which has an increased focus on quality and achievement of standards, assessment practices across mathematics departments may not always withstand scrutiny. Criteria based assessment is central to communicating to our students what is expected of their work and in demonstrating their achievements, yet its use in the context of undergraduate mathematics is not common. Even though the assessment tasks we give to our students may involve higher order skills and appropriate use of mathematical notation, these factors are not always adequately articulated to the students, nor are they reflected in the grading of the tasks.

This presentation will outline an Australian national project which aimed to engage mathematics educators, through several workshops, in a conversation about assessment. The project was designed to influence assessment practices in mathematics departments, to move away from idiosyncratic marking and grading approaches that favour procedural mastery, towards practices that focus on improving student learning and the certification of their learning. We will report on (i) the key learnings from the conversation with mathematics educators, (ii) the development of resources to support criteria based assessment, and (iii) the key findings from trials of those resources.

Mathematicians & Mathematics Education: A Marriage of Convenience in Professional Development

Greg Oates, Tanya Evans

University of Auckland, New Zealand

Previous Delta presentations have reported on the DATUM research project for professional development of mathematics lecturers at the University of Auckland (see e.g. Paterson, Thomas & Taylor, 2011; Paterson & Evans, 2013). The research project was theoretically grounded in Schoenfeld’s (2010) resources, orientations, and goals (ROG) model of teacher action and involved a group of mathematicians and mathematics educators, who met regularly to discuss and analyse video excerpts of their lecturing. These discussions were accompanied by written pre- and post-lecture statements of their “ROGs”. The trial proved successful enough to be expanded into
further groups that now constitute an ongoing professional development culture within our department. One of these groups has operated since 2012. It is very diverse in the nature of the courses they teach and their mathematical research interests, including algebra, analysis, applied mathematics and mathematics education at both the undergraduate and graduate level. A regular and interesting feature of the group’s discussions is the consideration of particular content items, and how we treat these within our different courses, in effect an examination of the epistemological and pragmatic value of these topics. This presentation will highlight examples of the content issues that have arisen at recent meetings, and consider the implications this has for examining and improving our overall teaching practices. We will consider the elements of the project that are considered essential to its ongoing success, and highlight the projects significance and implications for ongoing professional development in higher mathematics.

Improving Mathematics Knowledge of Educators in the Teaching of Statistics: A Case of Continuing Professional Development

Avhasei Richard Tsanwani

MASTEC Institute, South Africa

Teachers are expected to have a “deep understanding” of mathematical content knowledge and also be aware and competent in use of a range of alternative and appropriate teaching methods with an emphasis on meaning. In this regard, mathematics education majors have not been exposed to enough alternative teaching methods to be capable of teaching mathematics with an emphasis on meaning. This paper briefly discuss a twelve-week professional development workshop for Further Education and Training (FET) mathematics educators at MASTEC Institute. The aim was to help improve mathematics educator’s ability to teach mathematics concepts in Curriculum and Assessment Policy Statements (CAPS) with meaning. Data consist of a pre-and post-assessment of educators’ knowledge of the cognitive demands of mathematical tasks from the professional development sessions. Results of the study revealed that the professional development workshop did improve their knowledge of new FET challenging mathematical tasks and confidence in different Statistics topics. Implications and recommendations for professional development in mathematics are provided.

Tsitsikamma D1: Math educators

Curriculum mismatch: TVET colleges face major challenges
Batseba Mofolo-Mbokane

University of Pretoria, South Africa

The majority of South African youth do not manage to finish matric on time. If they do, they are not competent to pursue studies at institutions of higher learning like Universities and Universities of Technology. These students end up at Technical
Vocational Education and Training (TVET) Colleges. The vision of the White Paper for TVET colleges is: “to train young school leavers, providing them with the skills, knowledge and attitudes necessary for employment in the labour market” (White Paper 2013: 11). TVET colleges are therefore seen as pathways to address post school qualifications for these underprepared or the out of school learners. However, the curriculum offered at TVET colleges is generally unattainable by such learners. They joined TVET colleges with the hope of addressing skills and moving away from the academic field. Based on the interview from TVET college managers and lecturers, the enrollment for National Curriculum Vocational (NCV), which is highly funded by Department of Higher Education (DHET) are dropping and the results remain poor. Some of the reasons being that, NCV is hard for them or that NCV is not taking them anywhere, since after completing NCV or somewhere in the middle of the program such students end up enrolling for NATED courses which prepares them to be artisans. The aim of the research done was to enhance mathematics teaching and learning at NC(V) levels 2; 3 and 4 by developing professional learning communities at the TVET colleges. It is however evident that the main problem of drop in enrollments and poor pass rates at the TVET colleges, mainly with the NCV curriculum is curriculum mismatch. In this paper, issues regarding curriculum mismatch are deliberated on and suggestions are given. Concerns raised are based on entry routes into TVET colleges and exit routes out of TVET colleges for both NCV and NATED programmes.

Advanced mathematics curriculum for students preparing to teach middle school mathematics

Gary Harris

Texas Tech University, USA

In the US there have been numerous calls for increasing the rigor in the mathematics preparation of primary and secondary mathematics teachers. In this article we describe a rigorous three course curriculum that is intended to provide the future middle school math teachers with a deep understanding of the mathematics they will be teaching.

Opening Real Science: Engaging tomorrow’s Science and Mathematics teachers

Leigh Wood

Macquarie University, Australia

The Opening Real Science (ORS) project aims to deliver online courses using authentic teaching and learning content and cutting-edge cases to bring science and mathematics to life. The courses also highlight student misconceptions and thresholds in learning science and mathematics and address them using a series of activities and resources to educate across the spectrum from science phobics to advanced majors. The mathematics courses within the ORS project cover financial literacy, statistics, basic numeracy and mathematical modelling across three streams K-4, 5-8, and 9-12. Each course is 30 hours of work on key topics presented through discussion, reflection, and
application. These courses are currently being trialled in partner universities reaching over 50% of Australian undergraduate pre-service teachers. The feedback from these trials will be used to inform future designs using an iterative design based research and evaluation research.

Investigating how past experiences in mathematics have influenced pre-service primary teachers

Dilshara Hill

Macquarie University, Australia

This study looks at a group of students in Macquarie University studying to be primary teachers. Many students in this cohort have only basic mathematics skills and some have anxiety associated with mathematics, yet are intending to choose a career where solid mathematics skills are essential. Our study involved surveying these students regarding their past experiences and their attitudes to mathematics. We investigate their mathematical background by looking at what type of experiences they have had and whether they are positive or negative. Furthermore we look into whether these experiences have influenced the attitude that these students have towards mathematics. We aim to raise awareness about students’ attitudes and learning, while provoking thought about teaching the subject of mathematics.

Tsitsikamma D2: Technology

Mathematica, my teaching assistant

Esmé Voges

Tshwane University of Technology, South Africa

Classes at tertiary institutions are getting bigger, while students seem to be less prepared for independent studies in, especially, Mathematics. It is an even bigger challenge for students who enrol for a postgraduate degree several years after completing their last course in Mathematics. One possible way to assist underprepared students may be with tutorial classes, organised and supervised by the relevant lecturers. Students also often struggle with what they consider abstract concepts. Such concepts might be illustrated using an interactive demonstration. For example, students might be able to solve the spring-mass model but struggle to interpret the solution and/or to visualize the role of various factors such as damping and the spring constant. Mathematica can assist in both cases by giving a student an interactive tutorial with several questions together with an interactive demonstration of the role of the quantities in the spring-mass model. "A picture is worth a thousand words." In this presentation several examples of Mathematica as independent "tutor" will be demonstrated.
Developing interactive applets with GeoGebra: processes, impacts, technologies

Anthony Morphett, Sharon Gunn, Robert Maillardet

University of Melbourne, Australia

With continuing technological improvements, it is increasingly easy to produce high-quality interactive online resources to support learning of mathematics & statistics concepts. Software like GeoGebra (www.geogebra.org) allows users to produce dynamic, interactive constructions, with relatively low technical demands and moderate time and resource requirements. In this paper, we describe a project at the School of Mathematics & Statistics, The University of Melbourne, to produce a collection of interactive applets to enhance teaching in a range of undergraduate subjects offered by the school. The applets target specific learning & teaching needs of our school and are tailored precisely to our local learning & teaching context. We will give an overview of the project, and describe the approach taken by the project to identify teaching needs that could be serviced by new applets, and the design and feedback processes used to produce the applets. We give a preliminary survey of the outcomes of the project and discuss its impact on teaching & learning, both directly on the student experience as well as on academic staff through professional development. We offer a discussion of the technical merits and drawbacks of GeoGebra as a technological platform for the development of interactive applets for undergraduate mathematics and statistics, including data on the development time required to produce applets using GeoGebra.

Teaching Mathematics in the PC Lab – The Students’ Viewpoints Today and Five Years Ago

Karsten Schmidt

Schmalkalden University of Applied Sciences, Germany

The compulsory course in matrix algebra at the Faculty of Business and Economics at Schmalkalden University has been taught in the PC lab for a long time (setting: up to two students in front of a PC, instructor’s PC connected to a projector). The Computer Algebra System DERIVE is used throughout the semester. Students can install DERIVE on their private PCs and have access to DERIVE during the final exam in the PC lab (then, naturally, only one student per PC). Other courses like Introduction to Mathematics, and Introduction to Statistics, are still taught in a traditional classroom setting (using blackboard, overhead projector, and pocket calculators). In Oct 2010, a survey was carried out to check whether the students prefer a traditional or a technology-based course, and how well they cope with the technology. In Oct 2015, a similar survey will be carried out to check whether students’ attitudes towards the use of technology in the teaching of mathematics have changed over time. Results from this survey will be presented and compared to the results of five years ago.
Tsitsikamma C1: Professional development

An Analysis of the Reasoning Abilities of Students in the Transition Period from Secondary to Tertiary Mathematics.

Trudie Benadé and Sonica Froneman
North-West University, South Africa

This article reports the results of an empirical study to determine the reasoning abilities of a group of first year mathematics students at entrance level at a tertiary institution. In the empirical study the questions in the first mathematics test written on tertiary level, as well as the students’ performances in the test, were analysed using Lithner’s framework. Questions in the test were categorised as imitative (memorised, guided and algorithmic reasoning) and creative (local and global) reasoning. Student performances were evaluated in accordance with these categories to give an indication of their reasoning abilities. The results indicate that these students experienced difficulties when answering questions that required creative elements, in contrast with questions that were based mainly on memorized or algorithmic reasoning.

The impact of assumed knowledge entry standards on undergraduate mathematics teaching in Australia

Dr Deborah King and Ms Joann Cattlin
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Over the last two decades, many Australian universities have relaxed their selection requirements for mathematics-dependent degrees, shifting from hard prerequisites to assumed knowledge standards which provide students with an indication of the prior learning that is expected. This has been regarded by some as a positive move, since students who may be returning to study, or who are changing career paths but do not have particular prerequisite study, now have more flexible pathways. However, there is mounting evidence to indicate that there are also significant negative impacts associated with assumed knowledge approaches, with large numbers of students enrolling in degrees without the stated assumed knowledge. For students, there are negative impacts on pass rates and retention rates and limitations to pathways within particular degrees. For institutions, the necessity to offer additional mathematics subjects at a lower level than normal and more support services for under-prepared students impacts on workloads and resources. In this paper, we discuss early research from the First Year in Maths project, which begins to shed light on the realities of a system that may in fact be too flexible.
Great Expectations: expectations and attitudes of first-year mathematics students

Leanne Rylands, Don Shearman

University of Western Sydney, Australia

We report on a study of three first-year mathematics subjects to probe students’ perceived ability in several areas of mathematics; their expectations and attitudes; and their mathematics background. Surveys were conducted in the first lecture at the start of semester to predominantly first-semester, first-year students.

Many students were mathematically poorly prepared for university study. Despite this, almost all students expected to pass their mathematics subject, including those who reported that their algebra, statistics, trigonometry and calculus skills were weak or non-existent. The failure rates in the three subjects were all over 30%.

We report on several other interesting observations from this survey, including the fact that about 45% of students reported “Okay” to “Excellent” calculus skills, despite the fact that almost 60% of the students had not done any calculus at school or, one assumes, elsewhere. This brings into question how much of the terminology that we take for granted is understood by students.

Overall, students’ attitudes to mathematics were very positive and they expected to gain knowledge and understanding from their mathematics subject.

These results have important implications for how students approach their mathematical studies, the time and effort they devote to them and whether they make use of any available mathematics learning support services.

Tsitsikamma D1: Math educators

Numeracy and Mathematical Skills of Pre-service Primary Teachers

Therese Wilson, Bronwyn Ewing, Daniel Crane, Charisse Farr

Queensland University of Technology, Australia

Concern over the skills of Australian teachers has led to the proposed introduction of pre-registration testing of beginning teachers in numeracy and literacy. While this proposal has been under discussion for some years, pilot tests have been implemented across Australia in August 2015. Where students are likely to enter programs without the required standards, universities are encouraged to provide extra support for skill development.

Queensland University of Technology’s STIMulate program provides support for learning in Mathematics, Science and IT and is available to assist pre-service teachers with the development of their personal numeracy and general mathematical skills. An optional diagnostic test was introduced to encourage first year Bachelor of Education (Primary) students to develop their own individualised learning plan to access help
from STIMulate’s Peer Learning Facilitators. Only 9% of the cohort of 200 chose to complete the test and fewer sought any assistance.

Students enrolled in a new Masters of Teaching (Primary) course were required to complete the same initial diagnostic test to inform an assessment piece for their unit. A follow-up test at the start of the following semester showed no improvement including in the strands of Number and Measurement which had been the focus of the initial unit.

This paper discusses the results of the diagnostic test as well as the engagement of students with the project.

Zimbabwean pre-service teachers’ responses to matrix algebra assessment items
Kazunga Cathrine and Bansilal Sarah

University of Kwa-Zulu Natal, South Africa

Linear algebra is often regarded as a challenging course to learn and to teach and has received much attention in mathematics education research studies. However there are limited studies that have been carried out in Africa. This qualitative study was exploratory in nature with the purpose of identifying trends in the performance of the participant students in selected items based on the concepts of determinants, inverse matrices and solution of systems of equations. Twenty seven pre-service mathematics teachers from a university in Zimbabwe who were studying a course in linear algebra participated in the study. Data was collected from the students’ written responses to an assessment instrument and analysed using document analysis techniques. The findings reveal that many students had low levels of engagement with the concepts and seem to have learnt off certain techniques which they applied to solve the problems. The study recommends that the university authorities should consider planning further support and interventions that would provide opportunities for deeper engagement with the concepts.

An investigation of Zimbabwean pre-service students’ conceptions of vector space concepts
Mutambara Lillias H.N1, Sarah Bansilal2

1Bindura University of Science Education, Zimbabwe
2University of Kwa-Zulu Natal, South Africa

Research shows that students generally cope well with the procedures for solving systems of linear equations but struggle to grasp concepts such as vector space and subspace, linear independence and spanning. The purpose of this study was to investigate undergraduate mathematics students’ understanding of linear algebra concepts such as vector spaces. Tall’s theory of three worlds of mathematics encompassing the embodied, symbolic and formal worlds formed the framework for this study. The study was carried out with 10 pre-service student teachers at a university in Zimbabwe who were enrolled in a Linear Algebra course. The qualitative
study was involved in carrying out an in depth document analysis of the 10 students’ written responses to nine items. It was found that students had a shallow understanding of background concepts such as sets and logic. Some could not distinguish between an element of a space and a vector space itself, while some confused the product of two vectors with the product of a scalar and vector. Other findings were that students often confused algorithms for solving systems of equations with the procedure for finding a basis for a solution space. The instructions of some items were also misunderstood by participants. It is recommended that students should be given opportunities that can help them develop a conceptual understanding of the concepts instead of learning the procedures by rote.

Tsitsikamma D2: Technology

Western Cape Maths4Stats training 2012 to 2014 – valuable or not?

Renette J Blignaut¹, Abduraghiem Latief¹, Ronell Jacobus¹, Retha Luus¹, Morne Lamont², Rechelle Jacobs¹

¹ University of the Western Cape, South Africa
² University of Stellenbosch, South Africa

In 2012, the Statistics and Population Studies department at the University of the Western Cape became involved in the Maths4Stats training project which was launched by Statistics South Africa. This project’s aim was to teach the statistical content in the new school Mathematics curricula to Grade 10 to 12 educators. This presentation will focus on how the training was conducted, discuss content taught and report on the outcomes achieved over a period of three years (2012 to 2014).

A questionnaire on background information was anonymously completed by each participating educator. At the start of each training day, baseline (prior) knowledge related to the topics taught on that day was collected by means of a pre-test and at the end of each day the gain in topic understanding was gauged by giving the same test as a post-training test. On the last day, educators evaluated the training and provided comments and suggestions for future improvements.

Results showed that a significant higher post-test mark compared to a pre-test mark was achieved on all four training days in 2012. In 2013, knowledge improved significantly on three of the four training days and in 2014, knowledge improved significantly on two of the three training days. In 2012 and 2013, the greatest mark improvement was noted when regression and grouped data was taught. In 2014, the greatest mark improvement was seen when the more advanced probability content was covered.

Educators commented very positively on the training experience which they shared with colleagues.
Using clickers in the mathematics classroom: novelty or necessity

Karin Bothma

University of Pretoria, South Africa

Large classes are a reality for most undergraduate lecturers at the Department of Mathematics and Applied Mathematics of the University of Pretoria. From an active learning perspective, successful learning is dependent on interactive learning activities engineered by the lecturer in order to engage students. Classroom response systems (or “clickers”) can be used as a tool to promote student-instructor interaction and student engagement in large mathematics classes. Students use a remote control device to respond to multiple choice questions projected on a screen. The display of the class distribution of answers on a screen is then followed by a discussion of the reasoning behind the correct answer. As with all technology, it is not the use of clickers itself that promote learning. At the center of this process is the effective design of a clicker question based on an underlying pedagogy. In this study we discuss the principles for effective use of clickers in the classroom and investigate the design and use of clicker questions in order to promote successful mathematics learning.

TUESDAY: Plenary session

Collaboration
THURSDAY: Keynote speaker
The Judy Patterson Speaker

Prof Maxine Pfannkuch, University of Auckland, New Zealand

Associate Professor Maxine Pfannkuch is in the Department of Statistics at The University of Auckland. Her research interests centre on enhancing secondary and introductory students’ statistical and probabilistic reasoning and conceptual understanding using dynamic visualizations. She has led research projects on statistical thinking, literacy, and inference. Currently she is investigating practitioners’ perspectives on probability modelling and trialling probability model tools with students. She was involved in the transformation of the secondary school statistics curriculum in New Zealand, is Co-Editor of the Statistics Education Research Journal and Co-Editor for the First International Handbook of Research in Statistics Education.

Visualizing chance in introductory probability

Summary on page 49.

THURSDAY: Multiple Stream Session 5

Tsitsikamma C1: Statistics

Predictors of success and failure in Statistics

Lyness Matizirofa

University of Johannesburg, South Africa

Background: The poor performance of students entering South African universities has been well documented in literature. There are many factors which have impacted on their study performance. This study will identify factors which lead to students failing statistics.

Methodology: A cross-sectional study was carried out in three purposively selected study sites at the University of Johannesburg. Random sample of 100 undergraduate students majoring in marketing, accounting and engineering was used. Semi-structured interviews were conducted by a trained interviewer. A pilot study was conducted to ensure validity and reliability. Descriptive and inferential statistics was conducted. Ethical clearance was obtained from the Research Ethics Committee at the University of the Johannesburg.
Results: The results of the study revealed that class and tutorial attendance have a significant effect on performance in statistics. The achievement of students is negatively correlated with low socio-economic status. Doing pure mathematics in high school was significantly associated with performance in statistics.

Conclusion and recommendation: There are various internal and external factors to the university that contribute to academic performance of students. Identification of predictors of student’s performance is useful in understanding the factors that render students vulnerable to failure and hence permit the identification of vulnerable students. Further research is needed to explore the problem on a large sample including a variety of factors. Since class attendance and doing mathematics in high school were significantly associated with performance. Recommendations on 80% attendance can be made mandatory and bridging courses introduced to fill the gaps in mathematical knowledge helps.

Influence of matric mathematics versus mathematical literacy in the success rate of an introduction course in statistics at TUT

Mrs SA Mouton, Dr I Louw

Tshwane University of Technology, South Africa

The Department of Mathematics and Statistics (DMS) presents an introductory statistics course to various qualifications on different campuses of Tshwane University of Technology (TUT). This paper focuses on students from the Faculty of Management Sciences that come from various study fields and take Statistics (STATS I) as a service subject over a year. From 2008 to 2014 the success rate for STATS I average 36% compared to a benchmark of 76.5% set for TUT.

A thorough investigation was done to determine what numerical skills students need to successfully complete STATS I. The sample of students in the research was stratified according to the matric subjects Mathematics(Math) or Mathematical Literacy(MatLit). The performance of the two strata were monitored and compared in 2013 to determine what influence the mathematical school subjects have on the success rate of STATS I. The matric symbol obtained in Math or MatLit, the marks of four semester tests, exam admission mark and the final mark of the students were included in the calculations for this study.

According to the results it was very clear that there is a huge difference in the performance of the two groups of students, even more than was expected. Only 26% of students who had MatLit in matric passed STAT I compared to the 74% pass of students who had Math. Students are not equipped with the necessary skills in school to successfully complete STATS I on tertiary level.
A regression model to predict student success in Statistics at TUT

PH Kloppers, SA Mouton

Tshwane University of Technology, South Africa

The Department of Mathematics and Statistics of TUT presents an introductory statistics course (STATS) to various qualifications on different campuses of Tshwane University of Technology (TUT). These students come from various study fields and take STATS as a year subject. The average success rate for this course is only 36%.

One possible way to reduce the large dropout rate is to identify at risk to fail students very early in the year and to have some remedial programs in place.

For a student to sit down in the exam for STATS a predicate a mark of at least 40% is required. Since all students do not qualify to sit for the exam and there is a very good correlation between the predicate and the final mark the predicate mark will be considered to predict student success.

The mark obtained by students in the first test and the Admission Point Scores obtained from the National Senior Certificate are very good indicators of success rate in STATS.

Using EXCEL, the data of the 2013 STATS students, after the exam was written, was used to build a regression model to predict the success rate.

This model was used to predict the success of 2014 students and it was found that the model predict the success rate very accurately.

Finally it is concluded that, by using the previous year’s results for STATS, the success rate of the current year’s students can be accurately predicted early in the year.

Identifying and Evaluating Threshold Concepts in First Year Statistics courses at a large university in South Africa

Andre Swanepoel, Johann Engelbrecht, Ansie Harding, Lizelle Fletcher

University of Pretoria, South Africa

In the teaching of Statistics, certain concepts are experienced as more difficult to comprehend than others. Misconception of such concepts while studying Statistics on the 100 level is problematic since it might prohibit the student from understanding and grasping the core concepts upon which the discipline is developed and will also influence the student’s future studies of the discipline since no proper holistic view of the inner mechanics of the different procedures and techniques nor the interrelatedness of the different procedures and techniques will be present. These concepts are referred to as threshold concepts where a threshold concept is a conceptual gateway that opens up a new and previously inaccessible way of thinking without which you cannot progress in the subject.

The purpose of this research is to identify the threshold concepts in 100 level Statistics at a large South African university in a three year longitudinal study and to also determine their levels of difficulty (which describes how troublesome the
Learning from experience: The realities of developing mathematics courses for an online engineering program

Diana Quinn, Amie Albrecht, Brian Webby, Kevin White

University of South Australia, Australia

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Rarely do university departments of mathematics redesign their basic mathematics courses. Through developing an online version of our associate degree in engineering...
in collaboration with Open Universities Australia, we redesigned the first in a sequence of five engineering mathematics courses. The online cohort proved different to our face-to-face experience. We embarked on a process of refining the unit using experiential learning and action research. The 13 week unit is delivered up to four times a year and this paper reviews the first 10 cycles of enhancements over 3 years and unpacks the layers of hypotheses underlying development decisions. Several category themes were identified with a focus on students, teachers and learning activities. Investment in online developments for mathematics can have multiple flow-on impacts for other teaching modes. Good curriculum design, regardless of environment, will always be a cornerstone of effective course development processes.

**Designing and using informal learning spaces to enhance student engagement with mathematical sciences**

Jeff Waldock

*Sheffield Hallam University, United Kingdom*

Student engagement, satisfaction and academic success is built upon a sense of belonging to a professional community that provides, amongst many other things, comprehensive support. This can be achieved through a culture of expectation and behaviour, the provision of appropriate support structures and the effective use of carefully-designed physical and virtual learning space.

Suitably-designed open learning space facilitates staff-student and peer interaction by supporting new patterns of social and intellectual behaviour (Oblinger, 2005). Incorporating a disciplinary focus in the design helps learners identify with that discipline and feel they belong to a professional community; together with a managed peer-support network, this creates a partnership learning community within which student engagement can flourish. For some years, we have observed our students gathering to work in open space close to staff offices. A cross-level supportive network began to develop naturally as a result and this informed our thinking when offered the chance to design a new learning space for Mathematics. The space was opened in 2015 and early indications are that expectations are being met, more students engaging proactively in group work outside taught sessions and feeling better supported by staff and peers. In this presentation I will describe these initiatives in more detail, outline how they have been designed to create an effective partnership learning community and report results from a survey as evidence for their success.
Expectations and Implementations of the Flipped Classroom Model in Undergraduate Mathematics Courses

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University of Northern Colorado, USA

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The flipped classroom model is being used more frequently in undergraduate mathematics courses. As with any new teaching model, in-depth investigations of both various implementation styles and how the new model improves student learning are needed. Currently, many practitioners have been sharing their implementations of this model. However, there has not yet been an investigation of the various implementations of the model to discern general trends in this movement. With this research goal in mind, we conducted a study exploring various implementations of the flipped classroom model by interviewing 19 faculty members who experienced using this model at 14 different institutes. Results indicate that participants had similar motivations for implementation; however, subsequent implementations were different. In addition, we share participants’ perspectives on (a) student learning of pre-requisite, procedural and conceptual knowledge, and (b) how this particular model promotes such knowledge developments. Finally, we provide suggestions for future implementations and research regarding this particular teaching model.

Tsitsikamma D2: Teaching and learning practices

Using the Creativity-in-Progress Rubric in an Inquiry-Based Introductory Proof Course

Gail Tang¹, Gulden Karakok², Milos Savic³, Houssein El Turkey⁴, Emilie Naccarato² and Brent Hancock²

¹University of La Verne, ²University of Northern Colorado, ³University of Oklahoma, ⁴University of New Haven

Implementing inquiry-based learning (IBL) in introductory proof courses has potential to deepen student learning, mitigate difficulties in proving processes, and encourage mathematical creativity. However, without an appropriate formative assessment structure, students can feel overwhelmed trying to adapt to this new teaching style while learning to construct proofs. To help students overcome some of their frustration, we implemented a formative assessment instrument called the Creativity-in-Progress Rubric (CPR) on Proving. This formative assessment tool is created to foster students’ mathematical creativity as well as to enhance their proving skills. The CPR has two main categories with subcategories: Making Connections (Between Definitions/Theorems, Between Representations, Between Examples) and Taking Risks (Tools/Tricks, Flexibility, Perseverance, Posing Questions, Evaluation of the Proof Attempt). The description of the behavior of each subcategory is provided in three
levels: Beginning, Developing, and Advancing. In this presentation, we will share the design of an IBL introductory proofs course, describe the CPR, and explain the ways the CPR was implemented in class. We will also report on results from a qualitative study conducted with four students who took the IBL course. The interviews focused on their observations of the CPR and how they utilized it on their proving. Preliminary analysis shows that student use of the CPR aided them when they were “stuck” on a proof. Students also reported that the CPR was useful in helping them make connections in order to prove a theorem.

Teaching group theory using Rubik's cubes

Claire Cornock

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Being situated within a course at the applied end of the spectrum of maths degrees, the pure mathematics modules at Sheffield Hallam University have an applied spin. Pure topics are taught through consideration of practical examples such as knots, cryptography and automata. Rubik’s cubes are used to teach group theory within a final year pure elective based on physical examples. Abstract concepts, such as subgroups, homomorphisms and equivalence relations are explored with the cubes first. In addition to this, conclusions about the cubes can be made through the consideration of algebraic approaches through a process of discovery. The teaching, learning and assessment methods are explored in this paper, along with the challenges and limitations of the methods. The physical use of Rubik’s cubes within the classroom and examination will be presented, along with the use of peer support groups in this process. The students generally respond positively to the teaching methods and the use of the cubes.

Numerical solutions for the vibrating bar eigenvalue problem: a student exercise

Stephan V. Joubert, Temple H. Fay and Michael Y. Shatalov

*Tshwane University of Technology, South Africa*

The classical vibrating bar model is a popular model that is regularly discussed as a “real-life” mathematical model. Unfortunately, it does not model “real-life” too well at high frequencies and thus numerous models (such as the Rayleigh-Love and Rayleigh-Bishop models) have been developed by various well-known mathematical scientists to improve on the predictions of the classical bar model. “Solving” the classical bar model analytically is technically quite daunting to many senior undergraduates and even novice graduates. Attempting to modify the classical model (as did Rayleigh, Love and Bishop) to enable it to “model reality” more closely, results in more technical detail being added to the model and hence to the analytical solution method (if it exists). Consequently such an exercise is rarely undertaken at the senior undergraduate or novice graduate level. We have developed a novel, easy numerical routine that
determines the eigenfrequency and eigenfunction of vibration for the fundamental and first overtone vibration of the Rayleigh-Love model. This is achieved by using the classical bar model to determine a “good initial guess value” for our iteration process. With the aid of a computer algebra system (CAS), our iteration routine (and even the derivation of the model involved) is readily understood by novice graduate students. Indeed, for a given model, the numerical solution method is easy enough for senior undergraduates to grasp. Understanding and being able to model and solve an advanced bar model using standard calculus and a CAS is a pedagogical gem.

**Projective geometry as an undergraduate course: a tour of the three worlds of mathematics**

Alice. M.W. Hui

*University of KwaZulu-Natal, South Africa*

The motivation for projective geometry came from the fine arts, when people tried to draw an image on a paper as it was seen by the eye. In graphical perspective, parallel lines in a plane intersect in a vanishing point. Based on this property, projective geometry can be defined axiomatically with "elliptic parallel" axiom: any two planes meet in exactly one line, or any two lines in a plane meet in exactly one point.

In most undergraduate curriculums, synthetic is often overshadowed by analytic geometry. In this talk, it is argued that projective geometry, with emphasis on axiomatic approach, should be offered as an undergraduate course to develop students’ deductive reasoning. Suggestions will be made on how to teach this course, based on the theory of the three worlds of mathematics by David Tall, namely the embodied world, the proceptual, and the axiomatic world. Textbooks and possible follow-up projects will be also presented.
THURSDAY: Keynote speaker
David Wagner, University of New Brunswick, Canada

David Wagner is Associate Dean (Graduate Programs) in the Faculty of Education at the University of New Brunswick. His interest in human interaction in mathematics and mathematics learning inspires his research, including: identifying positioning structures in mathematics classrooms by analyzing language practice, ethnomathematical conversations in aboriginal communities, and working with teachers to interrogate authority structures in their classrooms. He serves on the Nonkilling Science and Technology Research Committee, the International Committee of Mathematics Education and Society, and the editorial boards of Educational Studies in Mathematics and Mathematics Education Research Journal. He has taught grades 7-12 mathematics in Canada and Swaziland.

Mathematics, Culture, and the Responsibilities of Mathematics Educators
Summary on page 53.

THURSDAY: Plenary session
Achieving Sustained Positive Outcomes in Maths and Stats Education: An Efficacy study
Seshni Mala

Pearson South Africa

At Pearson, we define efficacy as a measurable impact on improving someone’s life through learning. We have dedicated ourselves to the pursuit of efficacy and improving learner outcomes.

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**THURSDAY: Multiple Stream Session 6**

**Tsitsikamma C1: Miscellaneous**

**Conceptual or procedural mathematics for engineering students – views of two qualified engineers from two countries**

Christer Bergsten¹, Johann Engelbrecht², and Owe Kågesten¹

¹Linkoping University, Sweden

²University of Pretoria, South Africa

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This study forms part of a collaboration project between universities in South Africa and Sweden in which we investigate whether the emphasis in undergraduate mathematics courses for engineering students would benefit from being more conceptually oriented than a traditional more procedurally oriented way of teaching. In this paper, we report in some detail from two interviews with professional engineers, selected to represent two different ‘poles’ of engineering work. The aim was to explore different kinds of arguments regarding the role of mathematics in engineering work, as well as some common across contexts. Both interviewees feel that conceptual mathematics is more important for engineering work, although the role of the procedural aspect was seen by one of the interviewees also to be important, but in a very intricate way.

**Optimization in first semester calculus: A look at a classic problem**

Renee LaRue and Nicole Engelke Infante

*West Virginia University, USA*

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[http://dx.doi.org/10.1080/0020739X.2015.1067844](http://dx.doi.org/10.1080/0020739X.2015.1067844)
Optimization problems in first semester calculus have historically been a challenge for students. Focusing on the classic optimization problem of finding the minimum amount of fencing required to enclose a fixed area, we examine students’ activity through the lens of Tall and Vinner’s concept image and Carlson and Bloom’s multidimensional problem-solving framework. We are particularly interested in students’ evoked concept images for the mathematical concepts that play a role in the construction of the function whose minimum is needed. Data analysis revealed several gaps in students’ understanding of the optimization process. We highlight the mathematical concepts that play a role in these gaps, focusing on variables, function notation, function composition, properties of rectangles and the relationships between them, the role of the optimizing function, and the graphical representation of the function.

Tsitsikamma D1: Teaching and learning practices

Mathematics pre-knowledge for entering Economic and Management Sciences

Mardi Jankowitz, Ilsa Basson, Adele Immelman, Machtedl trydom

University of South Africa

The application of mathematics and statistics principles is very important in business and finance-related disciplines. A sound understanding of the manipulation of variables, handling of data and the analysis of graphs are examples of aspects that are essential for a good interpretation of “real-world” situations in the business world.

The University of South Africa recently introduced more stringent mathematics requirements for students entering various Economic and Management Sciences degrees. One of the questions debated was what the mathematics pre-knowledge level should be for a student to be successful at the first year of study? Would a minimum final mark of 60% for Mathematics in Grade 12 be essential to achieve success in the mathematics-based first level modules?

It was anticipated that an answer to the question could be found by considering the topics covered and examined in the final Gr 12 examination in the further education and training phase (FET) syllabus of Mathematics. The topics of the National Curriculum Statement (NCS) of 2013 and the 2014 Curriculum and Assessment Policy Statement (CAPS) were compared to the content of the syllabi of the mathematics-based first year level Unisa modules to determining possible gaps and overlaps.

Bloom’s Revised Taxonomy for Mathematics was used as a framework to classify each question of the final grade 12 Mathematics examination papers of 2013 and 2014 with a view of finding the level of the topics covered. The overlapping topics in the past examination papers of the mathematics-based first year level modules were similarly classified. A comparison of the levels of knowledge was made and it seems that a minimum mark of 60% for Mathematics in Grade 12 could predict success in mathematics-based first level modules.
Threshold concepts in finance: student perspectives

Susan Hoadley, Tim Kyng, Leonie Tickle and Leigh N. Wood

Macquarie University, Australia

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Graduates with well-developed capabilities in finance are invaluable to our society and in increasing demand. Universities face the challenge of designing finance programmes to develop these capabilities and the essential knowledge that underpins them. Our research responds to this challenge by identifying threshold concepts that are central to the mastery of finance and by exploring their potential for informing curriculum design and pedagogical practices to improve student outcomes. In this paper, we report the results of an online survey of finance academics at multiple institutions in Australia, Canada, New Zealand, South Africa and the United Kingdom. The outcomes of our research are recommendations for threshold concepts in finance endorsed by quantitative evidence, as well as a model of the finance curriculum incorporating finance, modelling and statistics threshold concepts. In addition, we draw conclusions about the application of threshold concept theory supported by both quantitative and qualitative evidence. Our methodology and findings have general relevance to the application of threshold concept theory as a means to investigate and inform curriculum design and delivery in higher education.
Tsitsikamma D2: Teaching and learning

Multicultural lecturing: some challenges

Jyoti Jhagroo

*Auckland University of Technology, New Zealand*

In this paper I discuss the cultural influences on mathematics education from my perspective as a teacher, and from recent migrant students’ perspectives on learning in a new country. I reflect on the assumptions I have made in my teaching and learning context that spans three decades, two countries (South Africa and New Zealand), one medium of instruction (English) and a shift from mono-cultural students to diverse multi-cultural, multi-national and multi-lingual students. Today, in New Zealand, there is no ‘elephant’ in classrooms, instead it seems to be a whole jungle. This is because my typical class includes: indigenous New Zealanders (Maori); immigrants from Europe (mainly from the UK); Pacific Islanders (Polynesians from Samoa, Tonga, and the Cook Islands); and more recently, immigrants (from South Africa, Asia, and occasionally South America); and I hear at least ten languages. What does this mean for me? Does it change my teaching? Have these students come to New Zealand to be indoctrinated into New Zealand styles of teaching and learning? These questions need to be considered from two perspectives, mine as lecturer, and from students as learners.

An Investigation into the Effect of Mathematics Self-Efficacy and Mathematics Anxiety on Mathematics Performance

Dr SJ Wagner-Welsh and Prof M Watson

*Nelson Mandela Metropolitan University, Port Elizabeth, South Africa*

Widening access has led to mathematically diverse first-year service course student populations at our university. Some first-year students lack essential conceptual knowledge and perform poorly in mathematics. This small pilot study investigated the existence of a relationship among mathematics self-efficacy, mathematics anxiety and performance in an effort to identify factors impacting on students’ mathematical engagement. A voluntary paper-based questionnaire was administered to a group of first-year students. A small sample size limited the significance of the results obtained. A weak correlation was found among mathematics self-efficacy, mathematics anxiety and performance. In general the results confirmed that students who obtain high mathematics marks have higher mathematics self-efficacy and lower mathematics anxiety levels. Students with higher mathematics anxiety levels performed more poorly in mathematics compared to students with lower anxiety levels. No differences in the mathematics anxiety levels of males and females could be found. The data did however show females have lower self-efficacy levels than males.
FRIDAY: Keynote speaker
Prof Joni Meyer, State University at Campinas, Brazil

Professor João Frederico C. A. Meyer (Joni to his friends) entered the State University at Campinas (UNICAMP) back in 1967 as an undergraduate, and began teaching there in 1971. He completed his Master's Degree and Ph.D. while working as a teacher and joined the Applied Mathematics Department when it was formed, in 1973, as part of the Institute of Mathematics, Statistics and Scientific Computing (IMECC) of UNICAMP. Since then he has worked with Numerical Analysis, especially in the Biomathematics Group. In 1978 he was one of the founders of the Brazilian Society for Applied and Computational Mathematics wherein he participated both in the Scientific Council as well as the Directing Board (of which he is now a member again, responsible for Mathematical Education activities). Joni Meyer was a founding member of the Brazilian Society for Mathematics Education (SBEM) and was the Dean of the Institute of Mathematics, Statistics and Scientific Computing from 2001 to 2005. He was the president of the Latin-American Society for Biomathematics for the 2006-2007 period, during which he organized their biennial meeting at UNICAMP. Joni continues to assume undergraduate and graduate teaching responsibilities, as well as the tutoring of master's dissertations and doctoral theses. He has been the university Vice-President for Outreach and Community Affairs since 2012.

Mathematical Disciplines for Undergraduate (and Graduate) Mathematics and Statistics: Challenges for Cooperating and Operating with Social Needs

Summary on page 55.

FRIDAY: Multiple Stream Session 7

Tsitsikamma C1: Statistics

Mathematical Preparedness in the Quantitative Sciences: The QuADDS Project

Greg Oates

University of Auckland, New Zealand

It has become increasingly difficult to accurately assess students’ mathematical preparedness for university courses based solely on their school qualifications, and it has become increasingly apparent that many students in first-year quantitative sciences courses (e.g. Mathematics, Statistics, Engineering, Physics, Chemistry etc) are mathematically under-prepared for their courses. The courses often assume a certain level of knowledge, and have limited opportunities to measure if this is the case, or
provide assistance for those who may not have the necessary skills. This can be even more problematic for students from disadvantaged or minority backgrounds, definitely the suspicion for the disproportionate number of Māori and Pasifika students who regularly feature in the “tail” of mathematics courses at Auckland University. In 2015, we began the QuADDS trial (Quantitative Assessment and Diagnosis for Development in Science) to try and address this issue using the commercially available online Pearson product *MyMathLab (MathXL)*. The reasons for this choice will be explained in the presentation. We first developed a series of subject-specific online diagnostic tests to measure students’ mathematical preparedness for first-year mathematics and statistics courses. Students were then provided appropriate levels of additional support based on their identified needs, one of which was an automatic, individualised and independent online study plan generated by *MyMathLab (MathXL)*. This study plan was based on students’ performance and responses in the diagnostic assessment with further learning development quizzes also provided online. This presentation will describe the design of the project, and report on its initial findings.

**Teaching Statistics to Engineering Students: A Thirty Year Perspective**

Richard Wilson

*University of Queensland, Brisbane, Australia*

Approaches to teaching have changed considerably over the last thirty years. Many of these changes have been driven by improvements in technology, but, more importantly, they have been driven by the necessity to make course content more appropriate to the changing requirements of graduates. This is particularly so in the area of statistical modeling and analysis. In considering these changes, it is of interest to discern what the changes have been with regard to how material is presented, what is being taught, what is required for students to understand and how their understanding is assessed. Determining how these changes have had an impact, both positive and negative, on the training of students and the outcomes of that training is also of interest. These aspects will be discussed in the context of teaching a large engineering statistics course over thirty years.

A brief description of the changes will be given. These changes have included the introduction of the use of statistical software, printed lecture notes and lecture recordings, and online tools for students to complete tutorial reports focused on analysing data. The response of students to these changes, especially with regard to learning outcomes, will be discussed. As well, the different learning styles which apply to these students will be considered and whether current methods and the predicted changes to these over the near future will enhance (or not) the learning experience and outcomes for these students. A key aspect will be the challenge of engaging these students with the course.
First level statistics students’ performance in a large classroom environment under the magnifying glass
Fransonet Reyneke
University of Pretoria, South Africa
In the past decade various intervention strategies regarding the traditional teaching model have been adopted by the Department of Statistics at the University of Pretoria in South African to address the problem of low success rates of its first level Statistics students.

Since 2012 we embarked on the challenge of substituting the traditional teaching model by a hybrid or blended model of the flipped classroom. In this flipped classroom environment, the transfer of information takes place online, as well as by pre-reading and using the Aplia interactive online-homework system. This approach allows for more quality time to be spent interacting with students inside the lecture hall, thus enhancing the deep learning process which takes place when students learn by knowledge scaffolding.

Students are no longer perceived as empty vessels that should be filled by lecturers. They want to be active and engaged in the lecture hall, which is a challenge in large classrooms. In 2015 we introduced the QT Clicker, not only to use for engagement in class, but also to write their tests and exams with. A cloud-based system called MindTap was also introduced in 2015. MindTap combines all the learning tools a student needs, into a singular learning path. The impact of the new interventions on the students’ performance and how the students perceive the changes in class, will be discussed.

Tsitsikamma D1: Miscellaneous

Oscillator Equations: An Energy Perspective
Temple H. Fay
Tshwane University of Technology, South Africa
Using a computer algebra system such as Mathematica, students in undergraduate mathematics, physics and engineering are able to investigate new and deeper mathematics and discuss more qualitative questions rather than simply solve exercises. As a case in point, arguably one of the most important topics in beginning differential equations is that of oscillator equations. These equations are ubiquitous in applied mathematics, physics and engineering. How- ever little is done in beginning texts on nonlinear models and almost nothing on how to compare oscillator models; and, in particular, almost no consider- ation is given to damping other than viscous damping. With the advent of sophisticated mathematical software such as Mathematica, solving numerically nonlinear models is virtually effortless, but his still leaves open the analysis of the various models themselves.
In this talk, we use an energy approach applied to damped oscillators. This damping may be quadratic, viscous, and a combination of these two called compound or more complex like that enjoyed by the Van der Poll Oscillator. This approach applies to nonlinear types of oscillators for which the pendulum is prototypical. Usually classical energy, kinetic plus potential, is expressed as a function of displacement and velocity. We propose an alternative that expresses this classical energy a function of displacement alone, bypassing velocity. We calculate the maximum amplitudes of oscillations without solving the model differential equation and are able to discuss the dissipation of energy. In particular, this yields a way to assess and compare models. We illustrate this through a compound damped pendulum model and the well-known Van der Pol Oscillator. We also suggest some student exercises.

**The impact of Mathematics on the Africa socio-economic takeoff**

Prof. Nouzha El Yacoubi

*University Mohammed V, Rabat, Morocco*

Irina Bokova, the UNESCO Director-General, said in her 2013 address: “…… to sustain Africa’s renaissance, science, technology, engineering and mathematics education (STEM) should lie at the heart of national development strategies……”. It is obvious that Africa can’t reach its goal of development and prosperity without focusing on Science, Technology, Engineering and Mathematics (STEM). At the heart of science and technology is mathematics! Mathematics are essential and critical, not only to the knowledge of Science, Technology and Engineering, but are also an important trump to take up the two big challenges of this new millennium: **innovation and competitiveness.** More, a study conducted recently (EISEM - May 2015, France) showed that for some developed countries, mathematics contribute up to 16% of GDP and 11% in terms of job creation. So learning outcomes in mathematics must be good. Unfortunately, the African students’ achievements in PISA and TIMSS are poor till now. This comes through some unsatisfactory educational conditions, but essentially through unqualified mathematics teachers. Some African countries have invested heavily in mathematics teacher initial training without succeeding to produce teachers with knowledge, skills and abilities enabling to form a foundation of schools of a good standard. Today, one of the major issues facing Education in the continent relates to the lack of relevant strategies to implement an efficient mathematics teacher continuous professional development in accordance with the high expectations of global innovation and advancements in Science and Technology that have strong relevance and potential impact for poverty eradication and the achievement of sustainable development.
Keynote speakers' summaries

The Rasch model for test outcomes and related item requirements

Prof Tim Dunne

University of Cape Town, South Africa

This paper is motivated firstly by a concern to open dialogues about testing mathematics (in particular) in various educational settings. All testing of persons is concerned with allocating a single summary mark as an adequate ordinal representation of the entire performance of each person in a test context. Equal marks suggest indistinguishable performances, and higher marks suggest superior performances, by intention of the test designers.

In mathematics settings, the presence and use of a marking memorandum (memo) implies that addition of allocated item scores is deemed a valid route to a single aggregated summary of test performance and hence, of ability.

Items may be dichotomous or polytomous, with open answer formats or closed multiple choice options. The memo encapsulates a notion of item difficulty within item responses, by allocation of scores under its explicit criteria. However, no mark or marks allocated using the memo criteria represent any absolute value or measure. Instead the memo scores of an item are simply an ordination of the typical expected responses to that item.

The primary question to be addressed is whether or not the device of adding scores across items can be corroborated as meaningful by the test data itself.

It transpires that there is only one family of statistical models which permits a sum of marks to be the single sufficient statistic for the notion of person performance in a test context. This family of models, called Rasch models, permits parameters for person performance and for measurement-like contrasts of performance. It also permits parameters for item difficulty and for measurement-like contrasts of item difficulty.

These parameters characterize stochastic models for test performance as item marks allocated to an observed interaction between an item in a test instrument and a test participant, in a specific context. There are profound education and ethical consequences.

The existence of a unique model which can characterize contrasts of difficulty and ability, is distinct from knowing whether or not the desirable model actually fits the data set at hand. Various specific assumptions have to be amenable to testing by data analysis, and discussion will focus on these complexities.

The paper has no new results, but seeks to present an accessible framework for debate of the Rasch approach as a sine qua non for the imposition of the ethical prerequisite for asserting measurement-like outcomes in education. The inferential questions that arise in the Rasch model framework address the matter of examining the extent to which patterns within
participant and item data in a test, fit a desired type of model, and if not, how the test data may be coherently but validly edited to achieve a clearer fit to the desired model type.

This reversed approach may sound ill-advised at first, by reason of its contrast with usual statistical practice of fitting models to data, and then iteratively selecting and including new parameters associated with other available explanatory information. The traditional objective is to explain pattern any variation in the data set by more complex or detailed models, until there is no apparent benefit from selecting further available information for an explanatory purpose.

As opposed to finding a model which merely describes the variability in test data of persons in some efficient way, the purpose of the Rasch model is to allocate locations on a common scale to both test participants and item difficulties, with measurement-like properties.

The distances between locations will be required to have strong fit with the observed interactions of persons and items, whose outcomes are recorded in the data. Specifically the distances between any person location and an item location should together fully determine the probabilities associated with all the permitted item scores in the test memorandum or rubric.

In particular, there should be no person variable or person factor, other than performance ability, associated with the parameter estimates. In contrast to classical methods, which seek to include and use explanatory variables, the Rasch approach seeks to establish that any available explanatory variables are unassociated with the person parameter estimates, after the item difficulty estimates have been taken into account.

By design, a test is required to operationalize the estimation of such person and item locations within a probability model. In the Rasch approach, the memo or the item may need to be reconsidered post hoc if their original form appears be less than adequate for estimating the outcome probabilities.

The basis upon which test data can be altered is that the nature of data is inherently ordinal, rather than interval or ratio measurement. The ordinality in test scores is expected to be internally consistent within each item and the scores on each item ought to be positively related to the total score on the remaining items.

A key insight into ordinal data adjustment is that any sum of item scores can be interpreted as an extended count or recount of elements along a sequence with intervals between consecutive intervals of arbitrary different lengths. Alterations of the item score data that preserve ordinal relations and remove inconsistencies and curiosities, such as apparent multiple modes, are legitimate in these circumstances.

The paper offers simple quick but instructive methods for exploring small test data sets, namely those data sets for which the number of persons writing the test is only a small multiple of the total of all discrete score elements in a marking memo.
In these contexts the Rasch estimates may be imprecise simply because the information in the small data set is too limited. The coherence of items necessary to permit an aggregation into a meaningful total can then be explored by the Gamma statistic for pairwise association between ordinal variables.

The Rasch methods presented in the paper will be most effective and relevant in large scale tests, for which the number of persons tested exceeds some ten to twenty times the maximal possible score count. It transpires that in the circumstances of large scale testing there are also likely to be more stringent ethical requirements for demonstrable comparability of performances and estimates of ability. Thus the importance of measurement-like properties is more critical in large scale tests than in small data set applications.

In the paper the descriptions will focus upon situations in which the observed data set is complete data, with all its elements observed and recorded. A substantial literature is available to handle estimation and inferences in situations in which one or more of the items have been been omitted by one or more persons and hence the corresponding elements of the data set are absent. The methods for incomplete data are not discussed directly in the paper.

The Rasch approach is not confined to mathematics testing, but can be used for any type of testing of purported competences within any educational discipline.

In fact the Rasch approach is also applicable to preference or intensity scales in the health and psycho-social sciences, where ordinal relations within items are aggregated into a single summative outcome. An extensive body of research literature is readily available.

In selecting and organizing this presentation I have drawn greatly on the immense contributions of David Andrich (UWA) to Rasch theory, to the software RUMM, and especially to their use and applications in education. I also acknowledge the influence and insight of Alan Tennant (now at Swiss Paraplegia) in his impacts of the emerging role of measurement-like explorations of preference and intensity data from questionnaire scales in the health sciences.

*Please see the conference Communications for the full paper.*
Think Big! A local initiative that became a National Network

Dr Deborah King

University of Melbourne, Australia

Over the past five or so years, Australia has experienced a much-welcomed resurgence of interest in tertiary mathematics education. Many factors have contributed to this movement including the personal interests and leadership of individuals, institutional changes that recognize the importance of university teaching, for example, the establishment of teaching-focused roles, and broad government initiatives which support the development of STEM education programmes.

In this talk I will describe my own evolution from research mathematician to mathematics education advocate, highlighting the challenges and barriers that I have encountered to that change and also the opportunities that have been afforded to me. Along the way I will discuss recent research projects and initiatives, some that I have been involved with over the last few years, and how these have helped to contribute to creating a network of mathematicians that have a significant interest in mathematics education.

In 2008 my appointment as Director of the Mathematics and Statistics Learning Centre at The University of Melbourne provided an opportunity to think more deeply about mathematics teaching. The focus of my work during this period was in first-year tertiary mathematics and although my interest in mathematics education was growing, research in education, or even scholarship of teaching and learning, was not considered to be legitimate research in my department. However in 2011, the university introduced new work-focus categories, including that of Teaching Specialist, which aimed to acknowledge the increasing diversity of modern academic roles. Taking advantage of this opportunity to change direction coincided with a cascade of events that have led to the most rewarding period of my academic life.

This transition could not have been made without the support and guidance of colleagues and of the networking opportunities provided by established communities of practice, like Delta. In fact, it was at Delta in 2011, that the seeds were sown for two national projects (funded by the Office for Learning and Teaching) Mathassess (led by Professor Cristina Varsavsky) and FYiMaths. Both of these projects have made significant contributions to the reinvigoration of national interest in mathematics education.

The FYiMaths project had two main goals; to build leadership capacity and raise the profile of individuals and teams coordinating and teaching first-year mathematics programmes, and, to develop a network of academics interested in promoting and supporting innovative approaches to learning and teaching in mathematics, particularly at first-year level.

From 2012 - 2014 the project team interviewed 40 academics across 26 universities in Australia and New Zealand and collected data from two national workshops. From this we were able to describe the typical role of an academic coordinating first-year tertiary mathematics programmes in Australia.
The findings of this project exposed significant deficiencies in the way academic coordination roles are designed, with little or no professional development for the incumbents and no clear parameters for their role provided via a position description. Based on this data, the team has produced a practical guide that includes multiple case studies and provides advice to Heads of School for designing a first-year mathematics coordinator’s position and also to academic staff in a coordination role or for those who may wish to take on such a position.

The project team was also able to document the common challenges that academics teaching first-year mathematics encounter. The major theme that emerged from our initial interviews was the challenge of dealing with under-prepared students.

Over the last 20 years, most Australian universities have relaxed their entry requirements for mathematics-dependent degrees, removing mathematics as a hard prerequisite. Instead, mathematics has become assumed knowledge, placing the responsibility on the student to ensure that they have met the mathematics standard required for their degree. As a consequence, the numbers of students entering universities without the appropriate level of mathematics has increased whilst the numbers of students studying intermediate and advanced level mathematics at secondary school has steadily dropped.

To explore the issue more fully, the FYiMaths team hosted a national forum, which brought together mathematicians, scientists, State education authorities and representatives from peak national bodies. Since then, we have continued to campaign for change, highlighting the negative consequences of the current practice.

The most significant achievement of the project is the FYiMaths network itself. In the three years since the network was launched, the informal membership has grown to around 200 members, with about 70 attending each workshop. The network is supported by a website that serves as an information hub and includes notices about upcoming conferences, positions vacant, and links to relevant articles.

Within the network, we have seen the development of a community of practice. From the beginning, there was evidence of strong interest from members in sharing their experiences and expertise with colleagues, which quickly led to the development of a sense of identity and recognition of a common cause which transcended institutional boundaries.

This young community has already made an impact on tertiary mathematics education in Australia through shared projects supported by successful national grants, cross-institutional collaborations and increased participation in national conferences. It has given a national voice to commonly held views in mathematics education and has the potential to affect change in a way that individuals acting in isolation cannot.

In addition to previously mentioned events, the network jointly ran the Connections and Continuity conference (with the Australian Association of Mathematics Teachers and the Australian Council of Deans of Science), which was the first of its kind to bring together mathematics educators from the secondary and tertiary sectors, to discuss transition issues.

The network is continuing to evolve, with the formation of state-based groups focused on local issues and forming important connections with the secondary school sector. It
has become an identifiable tertiary mathematics practitioner group within Australia, which has broadened its outlook beyond first-year issues to encompass undergraduate teaching more generally.

These advancements show that dissemination of local activities through networking and collaboration is a powerful force for change and the development of the scholarship of teaching and learning in undergraduate mathematics is a critical component in growing local initiatives to benefit the broader community.

References


Visualizing chance in introductory probability
Maxine Pfannkuch
University of Auckland, New Zealand

Introduction

Random events and chance phenomena permeate our lives and environments. Probabilistic reasoning is needed to help people operate sensibly and optimally in the face of uncertainty due to randomness in processes. The current approach to teaching probability however, particularly at the introductory level, draws on the tradition of classical mathematical probability, which may not be accompanied by a substantial understanding of the chance phenomena that mathematics can be used to describe. Often teaching approaches degenerate to a list of formulas and routine applications. Borovcnik (2011, p. 81) observed that, “probability is signified by a peculiar kind of thinking, which is not preserved in its mathematical conception.” Such a mathematical approach has resulted in many students unable to gain access to probabilistic ideas (e.g., Greer & Mukhopadhyay, 2005). Furthermore, the literature is replete with probability misconceptions observed in people’s thinking (e.g., Kahneman, Slovic, & Tversky, 1982; Konold, 1989; Lecoutre, 1992, Nickerson, 2004). The challenges and difficulties involved in teaching probability are well documented (Jones, 2005; Chernoff & Sriraman, 2014) as it seems to be a way of thinking that must be painstakingly taught through systematic education over many years (Fischbein, 1975). Researchers have recently proposed that a modelling approach to probability, with emphasis on modelling random behaviour, may be a fecund route for improving the teaching and learning of probability (e.g., Garfield, delMas, & Zieffler, 2012; Konold & Kazak, 2008)

With advances in technology and the ability to produce dynamic visualizations of phenomena, attention to visual representations is gaining momentum in education research (Mayer, 2010; Presmeg, 2006) as visualizations have the potential to allow new ways for students to engage with abstractions (Garcia-Retamero & Hoffrage, 2013). Access to technology has created an opportunity for students to experience random behaviour through simulations, to visualise chance through the creation of new representational infrastructure, and to gain access to concepts that were previously inaccessible (Sacristan et al., 2010). Limited research has been conducted on developing students’ probabilistic reasoning at the undergraduate level and on the use of visualizations within a modelling framework to enhance conceptual reasoning. Therefore we are currently conducting an exploratory study to understand what conceptual understanding needs to be and can be promoted with visualizations at the undergraduate level.

The Study

The core research team consists of two education researchers, three practitioners and two software advisors. The research method follows the principles of design-based research: identification and analysis of problematic areas, preparation and design of
tools and tasks, implementation with pairs of students, and retrospective analysis (Bakker & van Eerde, 2015).

To understand what concepts need to be promoted we interviewed seven practitioners, whose professional lives are centered on probability modelling over a diverse range of fields. From those interviews we determined the big enduring ideas that underpin probabilistic thinking and modelling, and the essential concepts. Based on literature reviews, including students’ probabilistic conceptions, our interviews with the practitioners, and a conceptual analysis of our targeted probability ideas, we designed and developed four software tools with accompanying tasks. The tools are dynamic visualizations that aim to visually represent concepts initially in a non-numeric format and to allow students to observe patterns and structure in the behaviour of phenomena. Three probability ideas that seem difficult for students are two-way tables of information (e.g., Watson & Callingham, 2014), independence (e.g., Watson, 2005,) and conditioning (e.g., Nickerson, 2004) including the confusion of the inverse and the base rate fallacy (Gigerenzer, 2014; Kahneman, 2011). For the former two ideas we designed an eikosogram, while for the latter a pachinkogram tool. To target the probability ideas of patterns of behaviour in randomness, distribution, and the connection to mathematical ideas we devised a tool for Poisson processes and a tool for Markov processes.

We trialled the tools and tasks on a number of first-year introductory probability students in order to ascertain the conceptual understanding and probabilistic reasoning being promoted. Working in pairs the students performed tasks using these software tools. Their thinking as they used the tools and their opinion on what they had learned was captured on audio- and video-tapes. As we trialled the tools and analysed the data in these pilot studies, we continued to modify the tools. Further modifications and additions to the tools are envisaged as we learn more about student reasoning processes and understand what conceptual understanding needs to be and can be promoted.

Findings

The main findings from the practitioners’ interviews were that seeing structure and applying structure were important aspects of probability modelling (Pfannkuch et al., 2016). A thematic analysis of the interviews produced four frameworks: (1) probability modelling approaches; (2) probabilistic thinking approaches to a problem; (3) a probability modelling cycle; and (4) core building blocks for probabilistic thinking and modelling. The core building blocks identified were: conditioning, distribution, randomness, and mathematics.

We are continuing to trial with students the four prototype tools, namely the eikosogram, pachinkogram, Poisson process tool, and Markov process tool together with accompanying tasks. Students’ interactions and reasoning with the tools are being analyzed. Currently our research findings indicate that the visualizations have the potential to enhance students’ probabilistic reasoning. The students expressed a wish that their instruction had incorporated these visualizations, as they believed that the visuals assisted their understanding of probabilities, conditioning, independence, the
link between waiting times and the Poisson distribution, probabilities associated with
states, the equilibrium distribution and the distribution of hitting times.

References

example from statistics education. In A. Bikner-Ahsbahs et al. (Eds.) Approaches to
qualitative research in mathematics education (pp. 429–466). Dordrecht, The
Netherlands: Springer Science+Business Media. doi: 10.1007/978-94-017-9181-6_16

In C. Batenero, G. Burrill, & C. Reading (Eds.), Teaching statistics in school
mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE
study: The 18th ICMI study, (pp. 71-83). New York, NY: Springer.


information improves diagnostic inferences in doctors and their patients. Social
Science and Medicine, 83, 27–33.

Garfield, J., delMas, R., & Zieffler, A. (2012). Developing statistical modelers and
thinkers in an introductory, tertiary-level statistics course. ZDM – The International

Viking.

Greer, B., & Mukhopadhyay, S. (2005). Teaching and learning the mathematization of
uncertainty: Historical, cultural, social and political contexts. In G. Jones (Ed.),


Heuristics and biases. New York, NY: Press Syndicate of the University of
Cambridge.

6(1), 59-98.

Innovations in Statistics Education, 2(1),
http://repositories.cdlib.org/uclastat/cts/tise/vol2/iss1/art1/.


Most definitions of mathematics are narrow. They point to certain practices associated with mathematics, they highlight abstraction, and they fail to mention human agents. Mathematics is much more than this. It is a human activity with a complex array of associated cultural practices. When we teach mathematics at any level, we face and listen to our students, and thus may become aware of the relationships at work in these contexts. However, it is also possible to turn our backs on the people in front of us and thus to position mathematics as free from cultural or individual expression. When we do mathematics outside the classroom, we have a similar choice. We can ground it in human problems or position it as independent and abstract. It is our choice to recognize the relational aspects of mathematics or to ignore them, and thus even ignoring them is a human choice.

In this address, I identify some of the cultural practices that connect to mathematics, and focus in particular on the way learners of mathematics may experience the discipline. First, I set the stage using a thought experiment and positioning theory to illustrate the way many discourses intersect with what happens in a mathematics classroom. These discourses (or stories) vary in scope from the very large, including various socio-economic and identity narratives, to the very small, including traditions unique to a particular classroom or even a small group. As suggested by positioning theory, it is instructive to focus on the actors in an interaction and only recognize transcendent entities, such as mathematics, as they are manifested through the people and artefacts in the interaction. This approach begs the question of how the discipline develops authority. Further, this question raises other questions about the role of mathematics teachers in constructing this kind of authority and thus constructing people’s experience of mathematics.

Second, I consider how roles, responsibilities, and expectations play out in classroom relationships. While I promote a view of mathematics that acknowledges the human agents in it, I recognize that part of the discipline’s value and its potential for social justice comes from its decontextualization. For example, when we add sums of money, measure goods for sale, design a structure, or execute governing practices, the results should not depend on the culture or position of the one doing the mathematics. Mathematics is supposed to be dependable and non-discriminatory. Mathematics can and should trump status-based power. The question is how the tension between abstraction and context might be negotiated in mathematics classrooms. This tension exists in primary, secondary, and tertiary contexts.

In mathematics, truth is supposed to come from justification, not from the status of the speaker in mathematics. Yet, I ask whether this is how students experience mathematics in their classrooms. To help with this question, I draw on my work with Beth Herbel-Eisenmann, in which we analyzed a large body of mathematics classroom transcripts. The analysis provides empirical evidence that the relationships among people in the classroom are strongly rehearsed in the interactions and that they favour a dependence on status or position. This is the opposite of what we might expect for a supposedly
rigorous practice like mathematics. In our analysis, we identified four authority structures in these mathematics classrooms, which we have claimed to be relevant to other mathematics learning contexts as well — personal authority, discourse as authority, discursive inevitability, and personal latitude. I consider how each of these authority structures or combinations of them can create for students a particular experience of mathematics.

There are other authority structures at play in addition to these four structures, though they could be described in terms of these basic structures. For example, there is an authority structure related to the mathematical valuing of mystery. In my current work with Annica Andersson, we examine this mystery discourse and consider the way authority works in mathematical practices that position some people as keepers of specialized knowledge and others as seekers. This mystery discourse appears to be pervasive in mathematics though it has two faces. Authority structures may also be situated in a particular discourse that intersects with mathematics classrooms. For example, partisan politics makes its way into mathematics classrooms through public discourse. I will show that teachers recognize a wide variety of authority sources in their mathematics classrooms. And again, I will ask how a mathematics instructor might acknowledge and deal with these authorities in tension with the kinds of truth claims that are supposedly inherent to the discipline of mathematics.

Third, I consider alternative approaches to mathematics instruction. I have often taken my classes on mathematical walks to observe the way natural phenomena inspire mathematical obsessions and how mathematical obsessions manifest themselves in human constructions. I have noticed how such a walk affects students’ orientation to otherwise typical mathematics classroom practices, though I have not researched this carefully. Both forms of mathematical obsession, linking to nature and to construction, have been referred to as mathematization. There are other ways to reimagine mathematics teaching, including critical mathematics and problem-based learning. Ethnomathematics, is perhaps the most well-known source of alternative stories of mathematics. I will share some of my ethnomathematical conversations with Indigenous elders in Canada to animate consideration of the possibilities and challenges for re-imagining mathematics teaching practices.

In the end, I aim to open up conceptualizations of mathematics to include diverse practices that may seem antithetical to mathematics. Recognizing the connections between these cultural practices and mathematics grounds questions that can guide reflective practice for mathematics educators. This reflection is by nature difficult because there are very many actors with a stake in what happens in mathematics classrooms. These tensions make mathematics what it is.
Mathematical Disciplines for Undergraduate (and Graduate) Mathematics and Statistics: Challenges for Cooperating and Operating with Social Needs

João Frederico da Costa Azevedo Meyer

State University at Campinas (UNICAMP)

At the very start, this presentation will mention differences in contexts and realities. This work will begin with two citations: one from Fernando Pessoa, a Portuguese poet and another by Saul Bellow, a North American novelist.

I will then try to mention possible means of studying answers to several questions, such as:

1. Do students share our enthusiasm with disciplines’ contents (and do we)? Why or why not?
2. Do we obtain students’ co-responsibilities in the learning processes? Why or why not?
3. Is what we teach (courses’ syllabus and classes) useful? And how so?
4. Where does Mathematical Modelling fit in? And how does it fit in?
5. Do we need a change of culture in teaching and learning as well as in teachers’ training? If we do, how to make it happen?
6. Are there relevant differences between Mathematical Modelling in school and in professional activities?
7. Why are technological tools so fundamental?
8. Do our students learn from us how to listen, create, criticize, accept boundaries, and mathematical and statistical daring?

My talk will obviously be biased by professional Mathematical Modellings in environmental problems, considering environmental mathematics as to include Society, Nature, Epidemiologies, Economics and Global Changes), as well as university teaching since 1971.
Summaries of posters

Students’ experiences of online assessment using Moodle
Shirley Wagner-Welsh and Pragashni Padayachee
Nelson Mandela Metropolitan University, South Africa

Assessments of large groups are one of the biggest challenges facing educators at many higher education institutions. Appropriate online assessment may address some of the challenges. In this research study the experiences of 392 students taking their assessments via Moodle at the Nelson Mandela Metropolitan University in the Eastern Cape of South Africa are described.

Preliminary analysis of the data revealed that respondents appreciated the flexibility of time and place that online assessment allowed them, the opportunity to learn independently, the opportunity to manage their own learning and the improved understanding of the subject matter afforded to them through the taking of practice tests and tutorials. Most negative aspects centred on network related issues and the user-friendliness of our Moodle sites.

Knowledge gained from this study may contribute to an improvement of the assessment practices of the MACC101 and the BBU1112 courses by presenting its difficulties and successes.

A mathematics blogging platform for Africa
Jonathan Shock
University of Cape Town, South Africa

In November 2014 during a workshop on mathematics communication in Africa at the African Institute for Mathematical Sciences, in Muizenberg, the idea of a new blogging platform was born. During the conference which included academics and teachers from over a dozen countries in Africa there were discussions about the problems with language issues surrounding mathematics teaching within Africa and about the lack of outreach activity to inspire and educate young students as to the relevance of mathematics to their lives.

It was decided that a website where anybody who wanted to write about mathematics in any African context, in any language, could be a great step in the right direction. And so Mathemafrica.org was born. The site is only a few months old, but we have around 30 bloggers signed up. We will soon be putting the framework of the site into more languages and are starting to get translations of blog posts too.

Eventually we would like to have bloggers from all over Africa writing about topics close to their hearts, be it a school student who has just come across an interesting idea in a maths class, to a first year varsity student who wants to talk about the shock of moving from matric to undergrad maths, to a researcher who wants to write about their
We believe that Mathemafrica can be a powerful platform for people to have their voices heard and for young people to be inspired to further look into the beauty of mathematics.

Real Analysis and GeoGebra: convergence of sequences
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This poster aims to present the role played by the software GeoGebra in an activity which the goal was to try an approach to teaching the notion of sequence convergence, more specifically to understand the relationship between $\epsilon$ and $n_0$ in the definition of numerical sequences. We use an experimental approach, in which some non-deductive methods are used, for example, intuition, visualization and/or analogy reasoning. We use the theoretical construct Humans-with-Media to emphasize that knowledge is produced by a collective of humans and non-humans, highlighting the role of the medium, in this case, GeoGebra. This is a qualitative study, in which we are concerned with a deep understanding of a particular social group: students-with-GeoGebra. In order to achieve the expected goal we used the Teaching Experiment Methodology, to explore the mathematics produced by students. The elements of the experiments are a sequence of teaching episodes consisting of an educational agent (researcher); one or more students and a recording device. We invited four students from an undergraduate Mathematics course in Brazil to participate in this activity. Its design allowed the students to elaborate numerical sequences with GeoGebra and, from the creation of sliding commands and a polygonal strip around the points of the sequences, to discuss the notion of $\epsilon$ and $n_0$. The results show evidence that GeoGebra contributed to develop a comprehension of the notion of convergence of sequences due to its dynamic nature and its visualization capabilities.

Impressions about teaching the course "Technological Resources"

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The aim of this work is to relate the conception and the teaching of one course that is part of the Postgraduate Program in Exact Sciences Teaching, a master degree level formation of the institution Centro Comunitário Univates, localized in Lajeado – Brazil. This course was designed to cover current elements in the teaching practice related to technologies, especially the digital ones. The motivation for such action, among various relevant, was the fact that most of the undergraduate courses at teaching does not address issues in technologies. Worried with this scenario the authors of this work developed a course with the objective of introducing it in the practice and think about your utilization. The structure of the course can be divided in three parts, wherein the first is realized a critical introduction and theoretical references are presented and discussed. The theoretical references consists of works of authors that make a reflexion of what is a good use of technological resources. The second moment is when softwares and online applications, covering themes in exact sciences, are introduced and explored with activities developed considering the theory. Finally, in the last
part of the course the students choose one online application or software to elaborate one activity and present it. After two editions of the course the authors have identified that students were motivated and engaged, and consider important to extend the use of technologies in other courses that they teach.